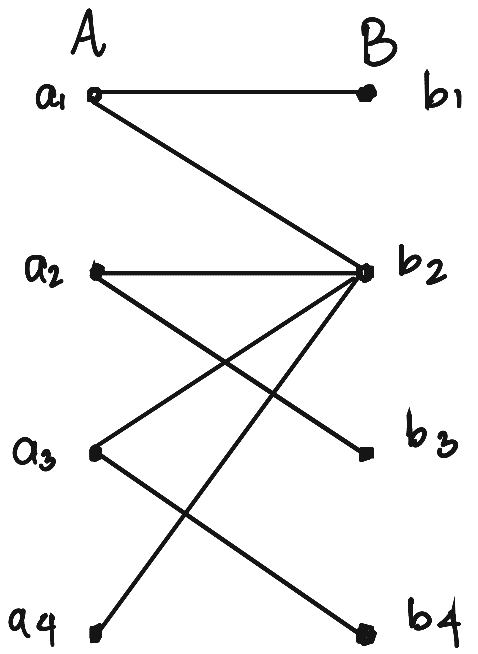
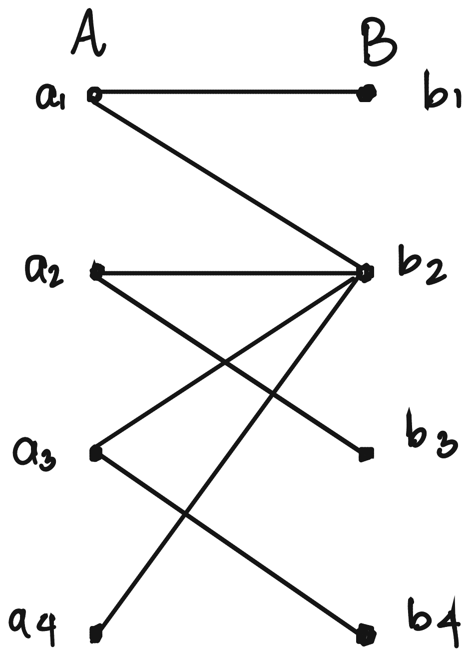


BIPARTITE GRAPH

$V = A \cup B$

BIPARTITE MATCHING M

A MATCHING M IS A SET OF EDGES SUCH THAT NO TWO EDGES IN M SHARE A COMMON ENDPOINT



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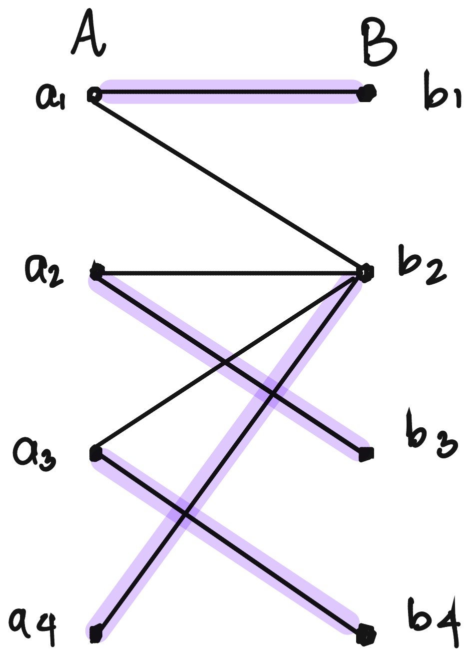
Q: FIND THE MATCHING OF MAXIMUM SIZE IN THE ABOVE GRAPH?



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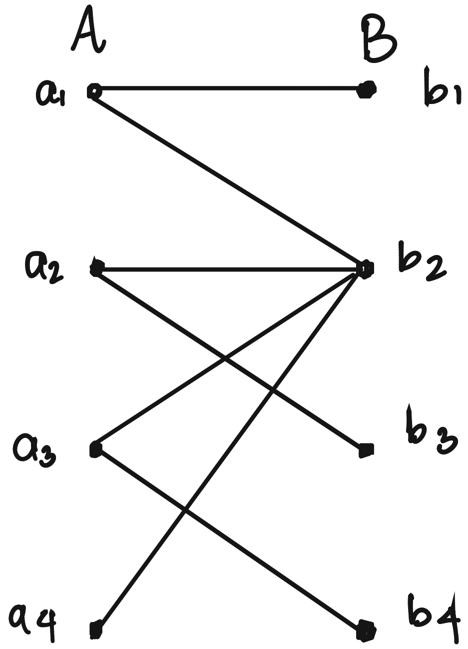
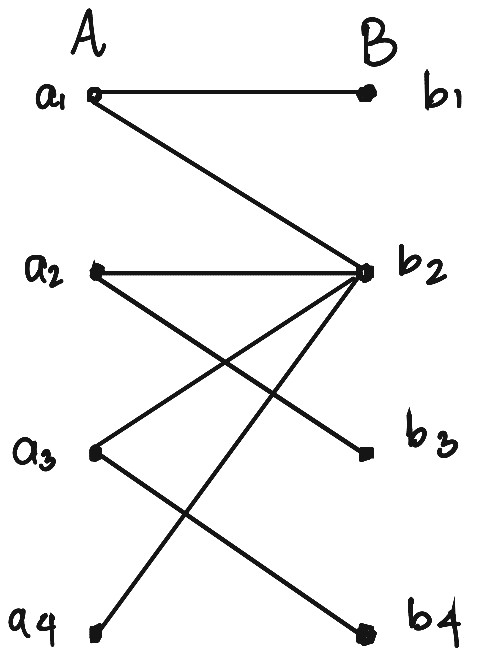
A: $M = \{(a_1, b_1), (a_2, b_3), (a_3, b_4), (a_4, b_2)\}$

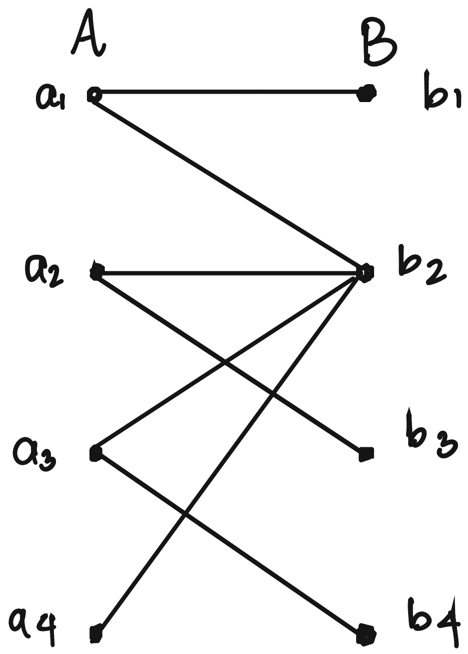


GIVEN AN UNDIRECTED GRAPH G, FIND A MAXIMUM MATCHING IN G.



GIVEN AN UNDIRECTED GRAPH G, FIND A MAXIMUM MATCHING IN G.

CONVERT INTO A FLOW PROBLEM:



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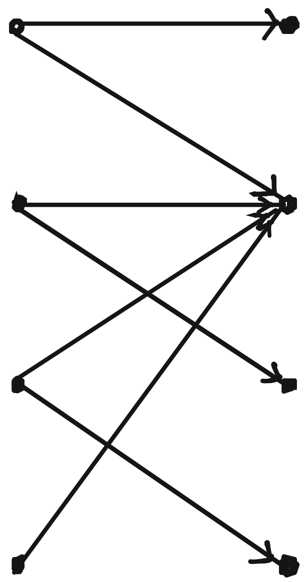
CONVERT INTO A FLOW PROBLEM:

→ DIRECTED GRAPH

→ s

→ t

→ CAPACITY ON EACH EDGE



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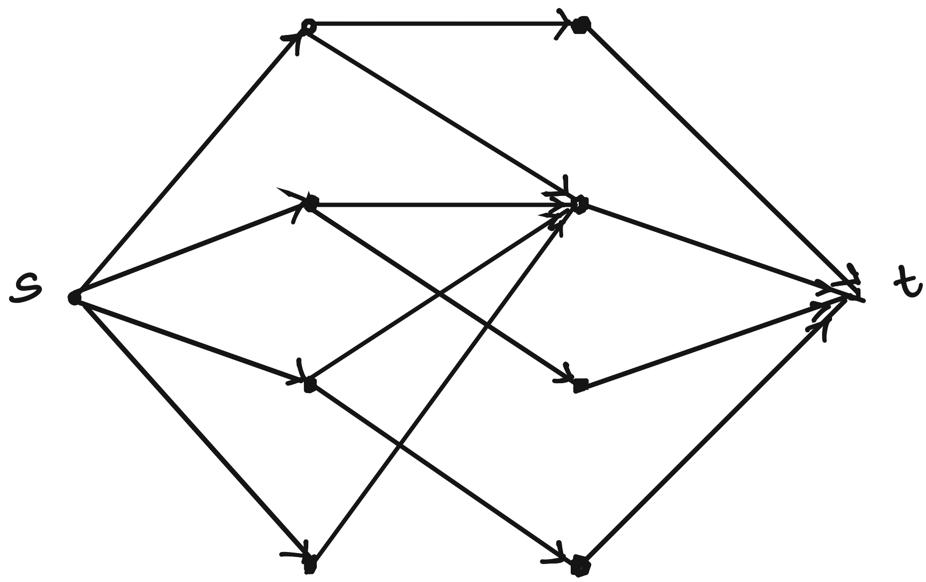

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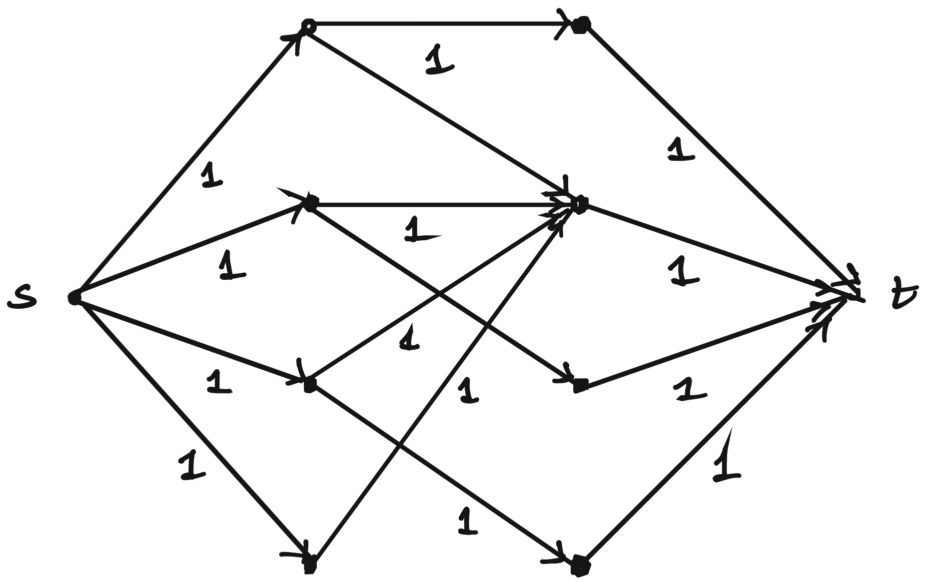

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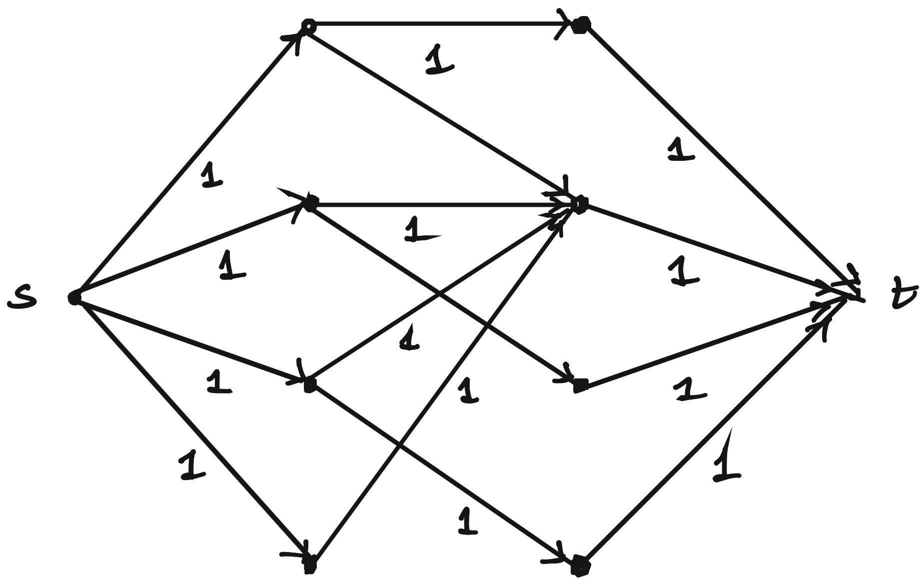
→ s ✓

→ t ✓

→ CAPACITY ON EACH EDGE ✓

CONVERTED TO FLOW PROBLEM.



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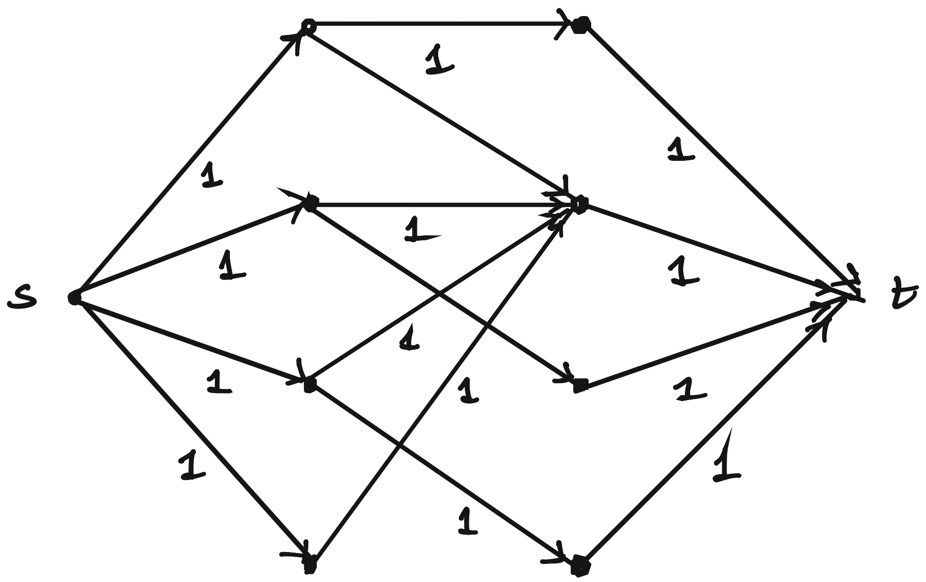
APPLY FORD-FULKERSON ALGORITHM



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→ s ✓✓

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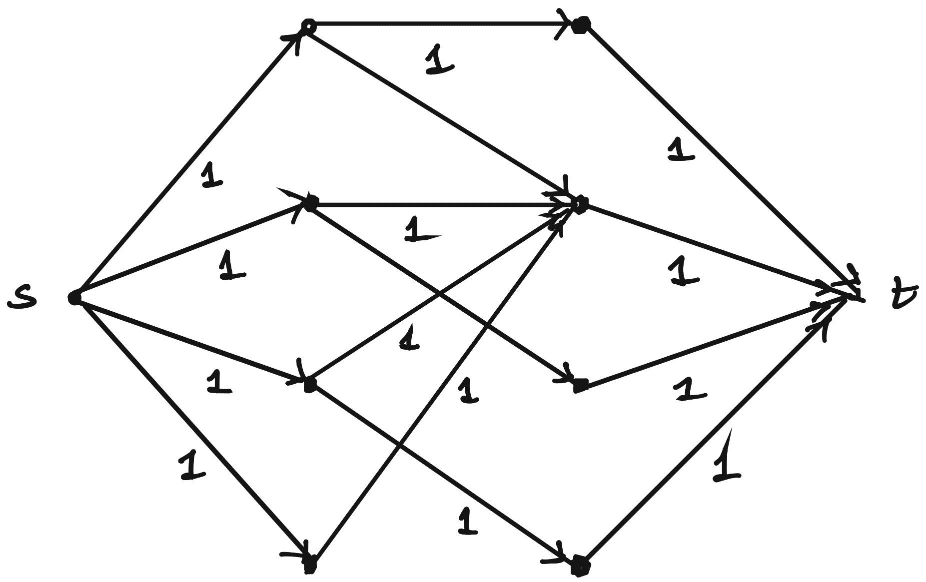
Q: WHAT FLOW VALUE YOU WILL GET IN ABOVE EXAMPLE?



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→ s ✓

→ t ✓

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CONVERTED TO FLOW PROBLEM.

APPLY FORD-FULKERSON ALGORITHM

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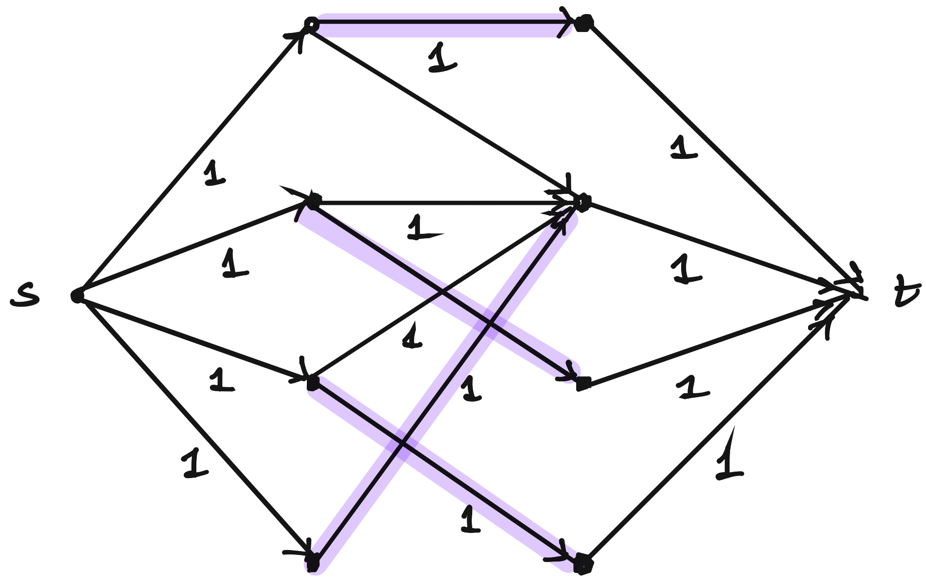
A: 4

$\Rightarrow$ IF THE MAXIMUM MATCHING IN G HAS SIZE K, THEN MAXIMUM FLOW IN G' IS $\geq K$.

⇒ IF THE MAXIMUM MATCHING IN G HAS SIZE K, THEN MAXIMUM FLOW IN G' IS ≥ K.



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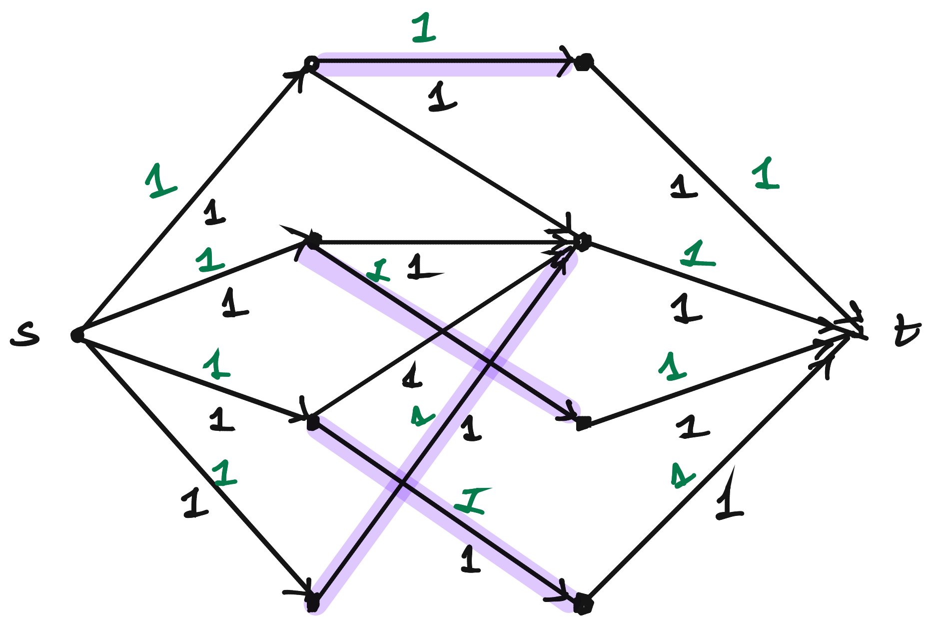

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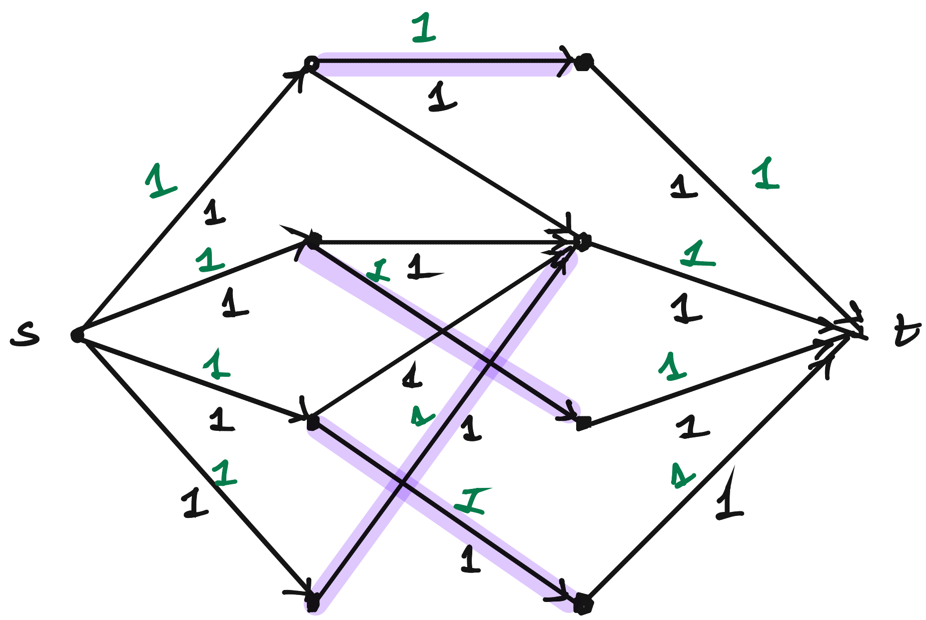
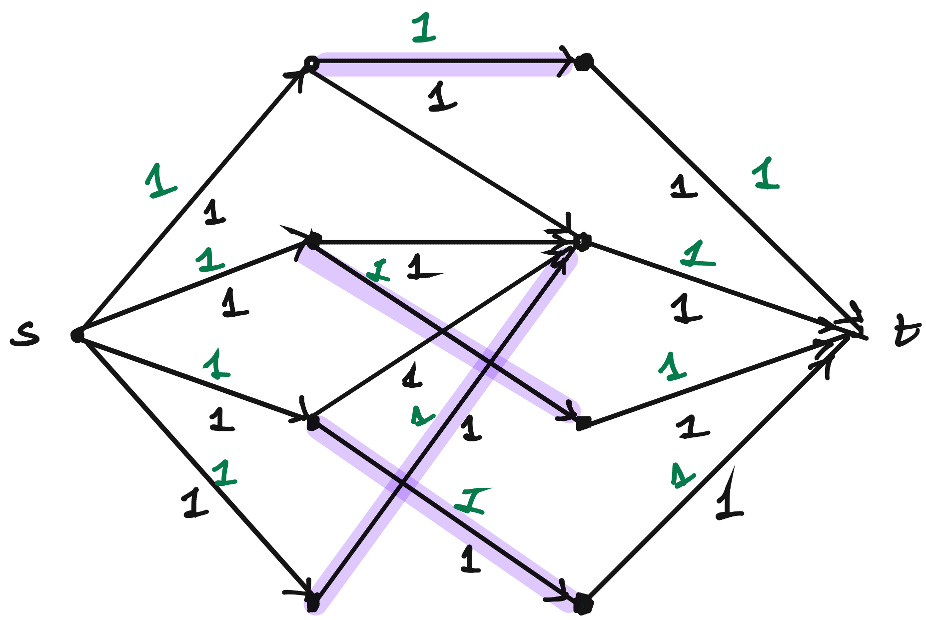


NEED TO SHOW

(1) CAPACITY CONSTRAINT

(2) CONSERVATION OF FLOW

⇒ If the maximum matching in G has size K, then maximum flow in G' is $\geq K$.



Need to show
(1) Capacity constraint
(2) Conservation of flow

Trivial.

$\Rightarrow$ IF THE MAXIMUM MATCHING IN G HAS SIZE K, THEN MAXIMUM FLOW IN G' IS $\geq K$.

$\Leftarrow$ IF THE MAXIMUM FLOW IN G' IS K, THEN MAXIMUM MATCHING IN G IS $\geq K$

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$\Rightarrow$ MAXIMUM MATCHING IN. G

$=$ MAXIMUM FLOW IN G'

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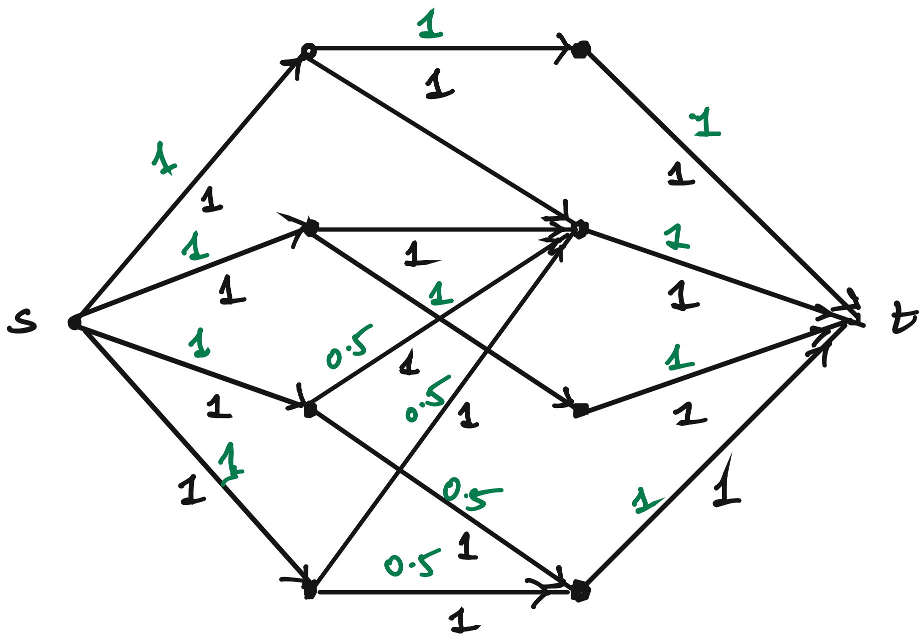
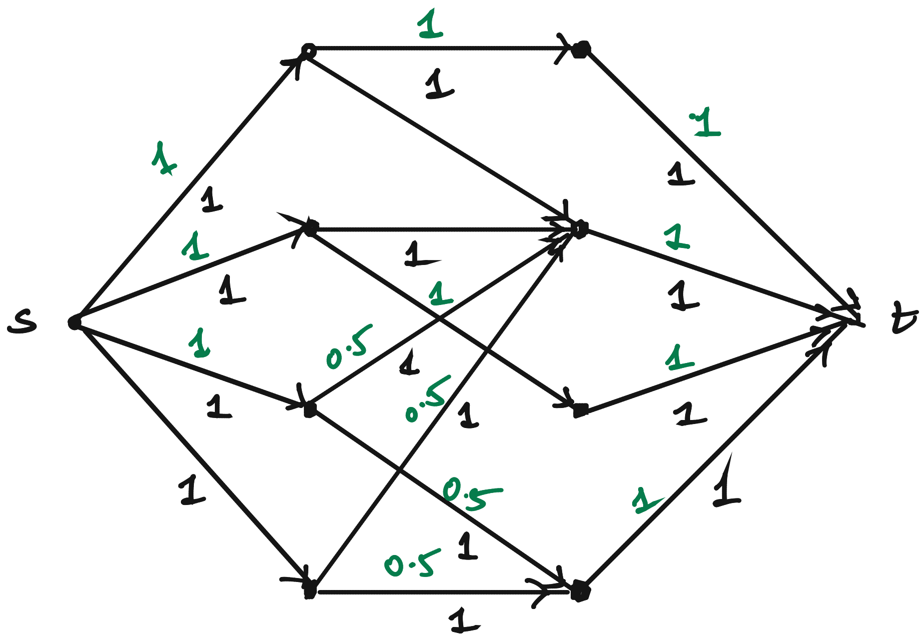
NEED TO SHOW THIS NOW

$\Rightarrow$ MAXIMUM MATCHING IN. G

= MAXIMUM FLOW IN G'



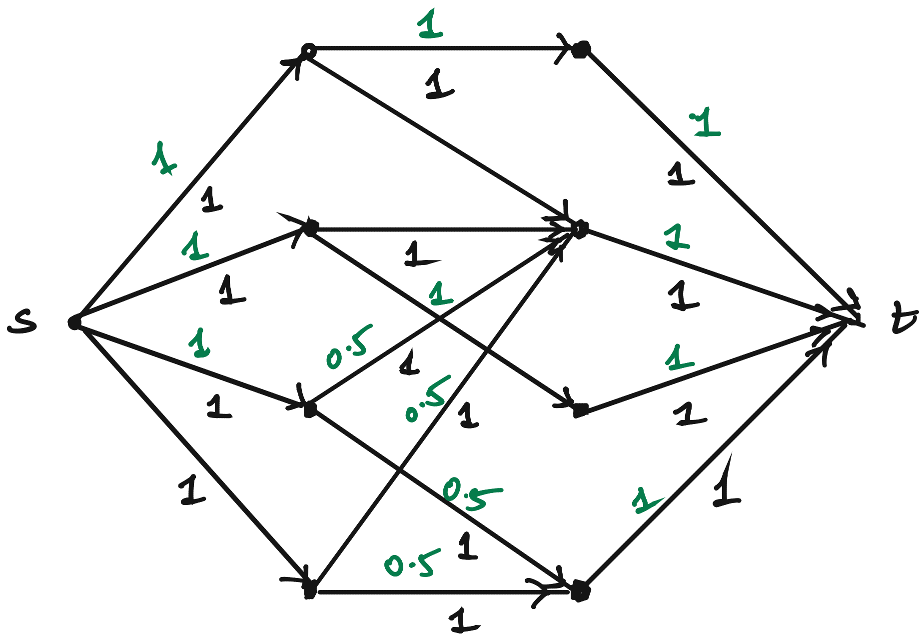
Q: CAN FF ALGO OUTPUT THE ABOVE FLOW?



Q: CAN FF ALGO OUTPUT THE ABOVE FLOW?

A: NO,

AT EACH ITERATION, FLOW THROUGH AN EDGE INCREASES OR DECREASES BY AN INTEGER "b".



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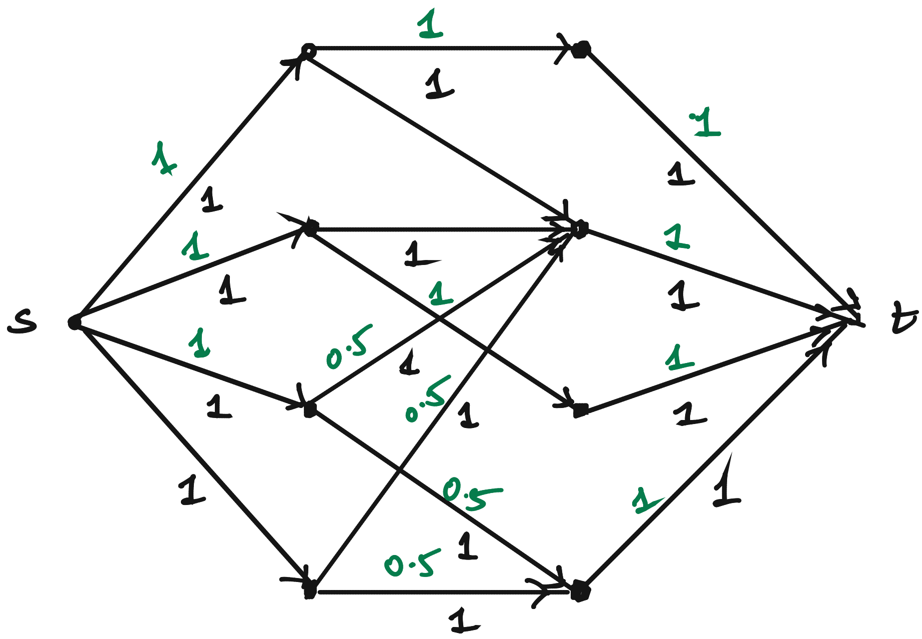
$f \leftarrow$ MAXIMUM FLOW IN G'

$$f(e) = \begin{cases} & \end{cases} \quad \forall e \in G'$$



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$f \leftarrow$ MAXIMUM FLOW IN G'

$$f(e) = \begin{cases} 0 \\ 1 \end{cases} \quad \forall e \in G'$$

FROM THE FLOW IN G', WE WILL NOW MAKE A MATCHING IN G.

$$M = \{(a,b) \mid a \in A,\ b \in B \text{ \& } f(ab) = 1\}$$

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$A^* = s \cup A$

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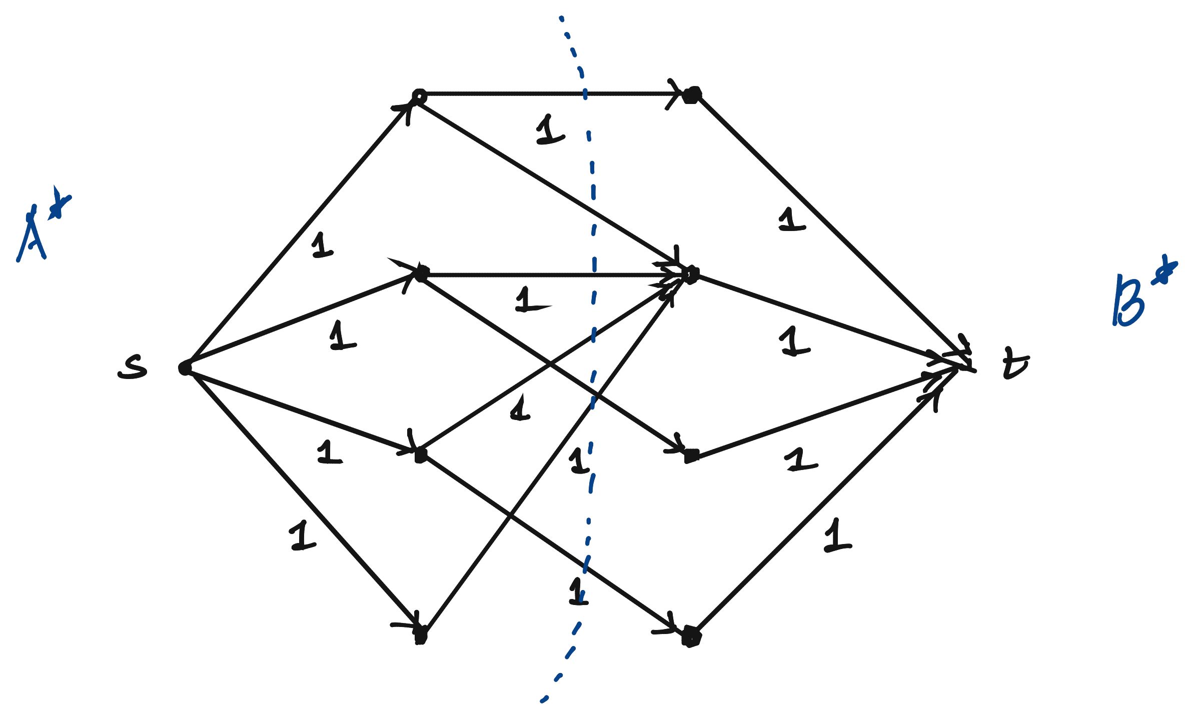

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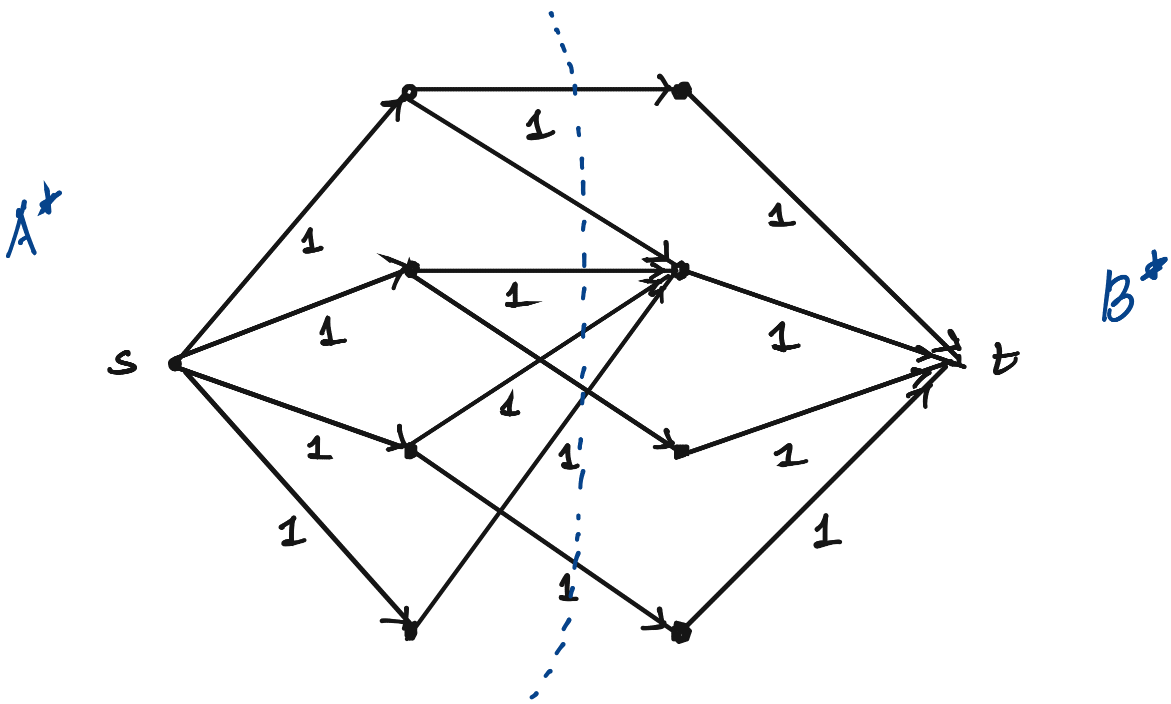

$$\text{FLOW FROM } s = \sum_{e \text{ OUT of } A^*} f(e) - \sum_{e \text{ IN } A^*} f(e)$$

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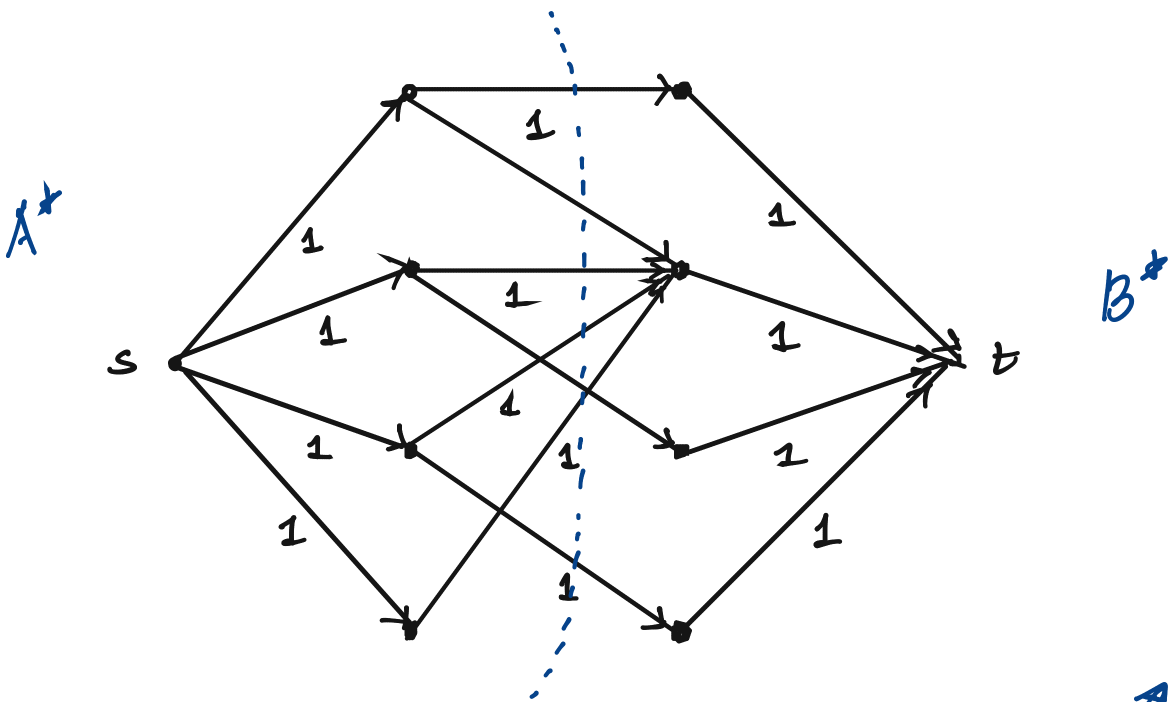

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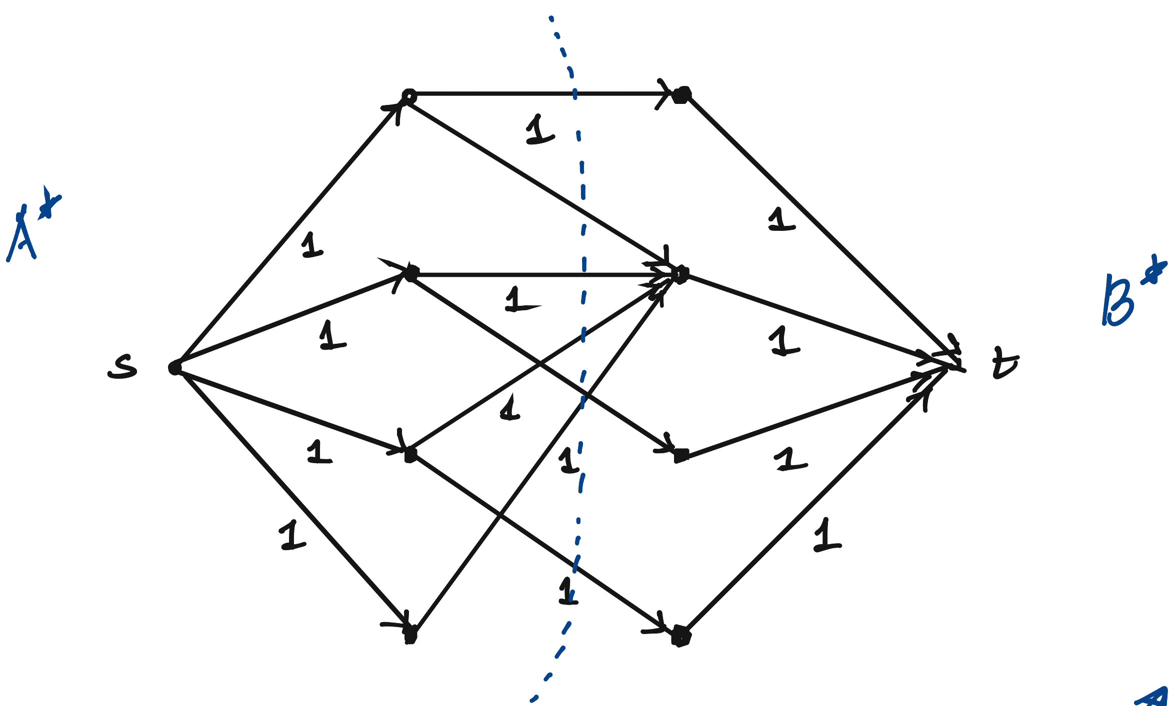

FLOW FROM s = $\sum_{e \text{ OUT of } A^*} f(e) - \sum_{e \text{ IN } A^*} f(e)$

LOOK AT ALL EDGES OUT OF A^* WITH $f(\cdot) = 1$ $\Rightarrow$ WE GET EXACTLY K SUCH EDGES.

FROM THE FLOW IN G', WE WILL NOW MAKE A MATCHING IN G.

$$M = \{(a,b) \mid a \in A, b \in B \text{ \& } f(ab) = 1\}$$

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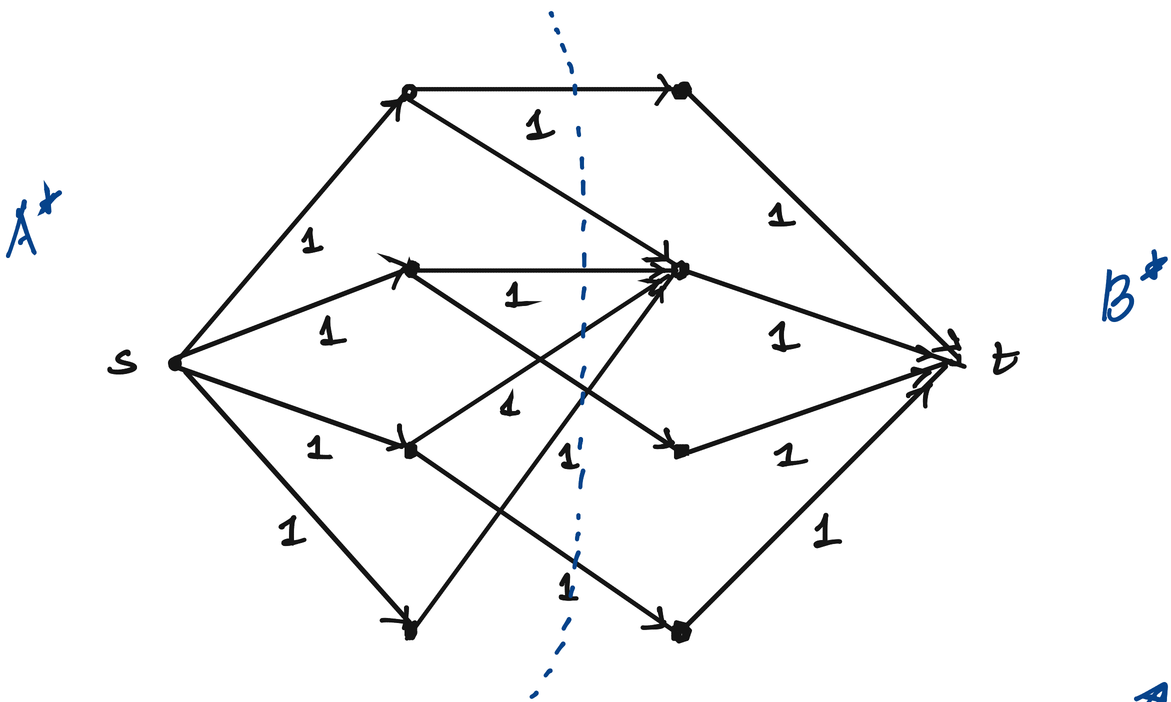

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IS THIS A MATCHING?

GIVEN A DIRECTED UNWEIGHTED GRAPH, s & t, FIND THE MAXIMUM NUMBER OF EDGE DISJOINT PATHS FROM s to t

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FOR FLOW

DIRECTED GRAPH

s

t

CAPACITY ON EACH EDGE

GIVEN A DIRECTED UNWEIGHTED GRAPH, s & t, FIND THE MAXIMUM NUMBER OF EDGE DISJOINT PATHS FROM s to t

FOR FLOW

DIRECTED GRAPH ✓

s ✓

t ✓

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LEMMA: IF THERE ARE K- EDGE DISJOINT PATHS IN A DIRECTED GRAPH G FROM s to t, THEN THE VALUE OF THE MAXIMUM s-t FLOW IS $\geq K$

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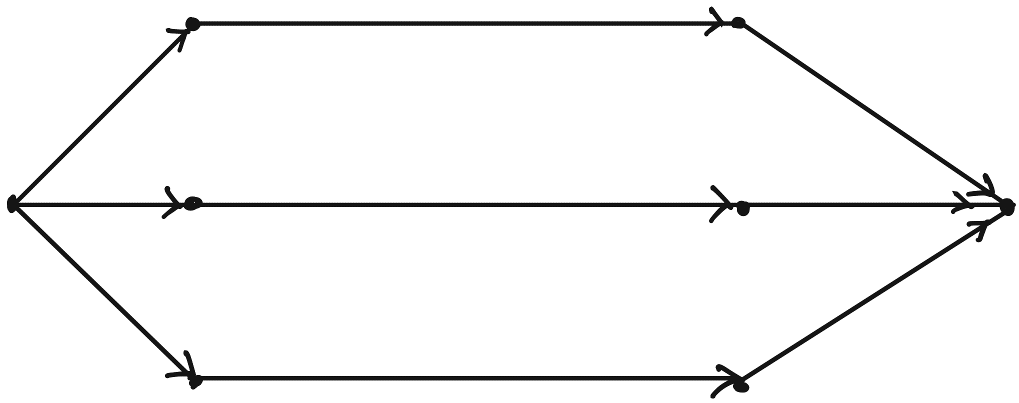
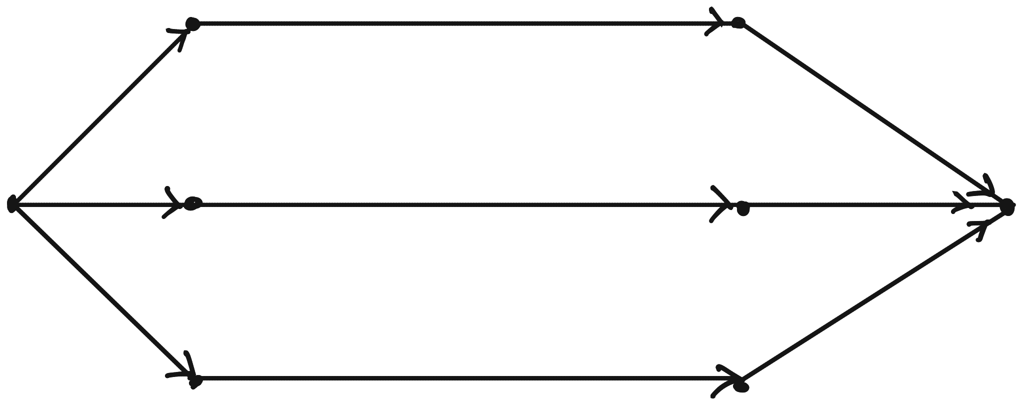


EDGE DISJOINT PATH = 3

Q: CAN YOU FIND A FLOW OF VALUE < 3

LEMMA: IF THERE ARE K- EDGE DISJOINT PATHS IN A DIRECTED GRAPH G FROM s to t, THEN THE VALUE OF THE MAXIMUM s-t FLOW IS $\geq K$



EDGE DISJOINT PATH = 3

Q: CAN YOU FIND A FLOW OF VALUE = 3

A: SEND A FLOW OF VALUE 1 ON EACH OF THESE PATHS.

LEMMA: IF THERE ARE K- EDGE DISJOINT PATHS IN A DIRECTED GRAPH G FROM s to t, THEN THE VALUE OF THE MAXIMUM s-t FLOW IS $\geq K$

$\Leftarrow$ **LEMMA** : IF THERE IS A MAXIMUM FLOW OF VALUE K IN G, THEN THERE ARE AT LEAST K - EDGE DISJOINT PATHS IN G.

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MAXIMUM FLOW IN G = MAXIMUM EDGE DISJOINT PATHS IN G.

OBSERVATION :

$$f(e) = \begin{cases} 0 & \forall e \in E \\ 1 & \end{cases}$$

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PROOF: BY INDUCTION ON K.

BASE CASE: $K = 0$

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PROOF: BY INDUCTION ON K.

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TRIVIALLY TRUE.

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PROOF: BY INDUCTION ON K.

BASE CASE: K = 0

TRIVIALLY TRUE.

INDUCTION HYPOTHESIS: IF MAX FLOW = K-1, THEN THERE ARE AT LEAST K-1 - EDGE DISJOINT PATHS

LEMMA: IF THERE IS A MAXIMUM FLOW OF VALUE K IN G, THEN THERE ARE AT LEAST K-EDGE DISJOINT PATHS IN G.

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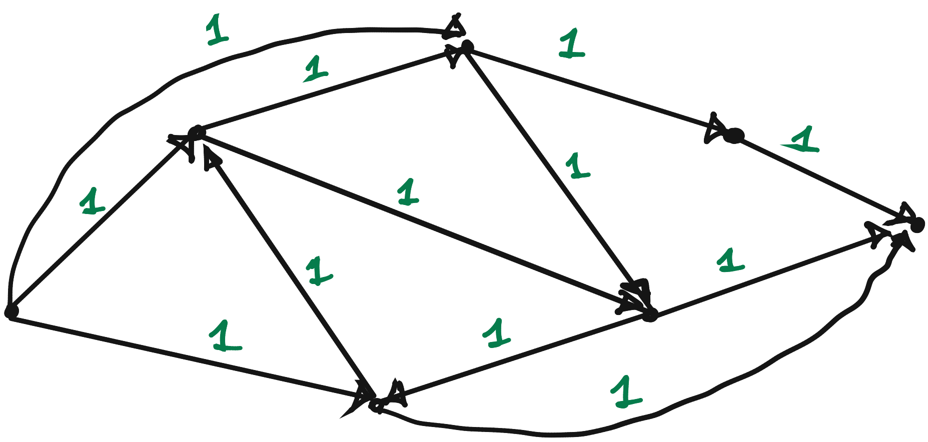
START WITH A FLOW OF VALUE K



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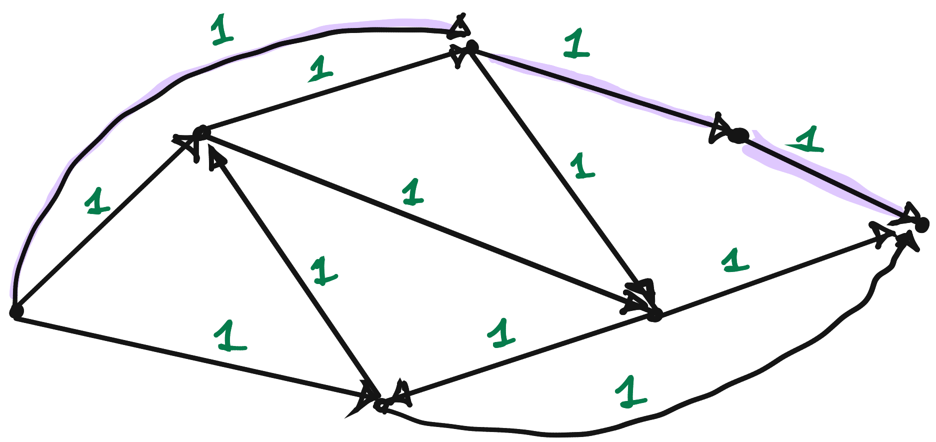
START WITH A FLOW OF VALUE K



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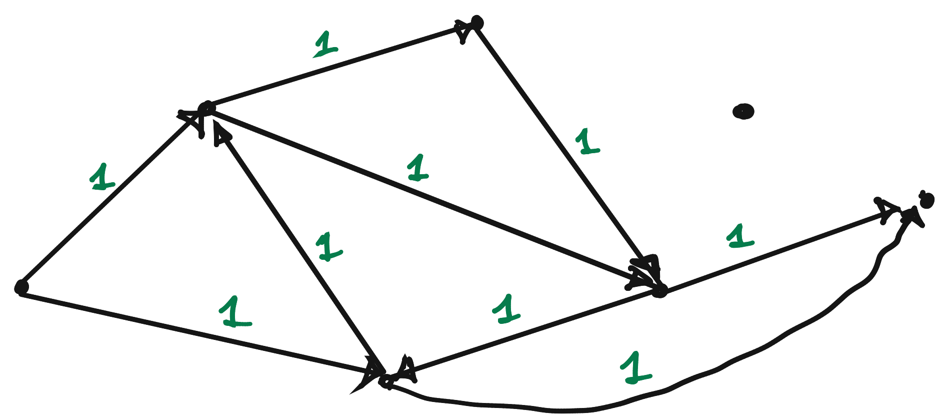


Q: FLOW IN THE RESULTING GRAPH ?

LEMMA: IF THERE IS A MAXIMUM FLOW OF VALUE K IN G, THEN THERE ARE AT LEAST K-EDGE DISJOINT PATHS IN G.

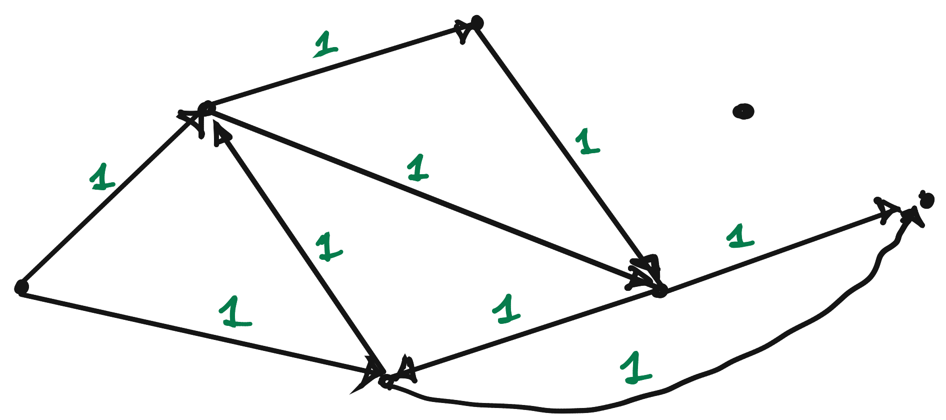
PROOF: BY INDUCTION ON K.

BASE CASE: $K=0$

TRIVIALLY TRUE.

INDUCTION HYPOTHESIS: IF MAX FLOW $= K-1$, THEN THERE ARE AT LEAST $K-1$-EDGE DISJOINT PATHS

START WITH A FLOW OF VALUE K



Q: FLOW IN THE RESULTING GRAPH?

A $K-1$

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TRIVIALLY TRUE.

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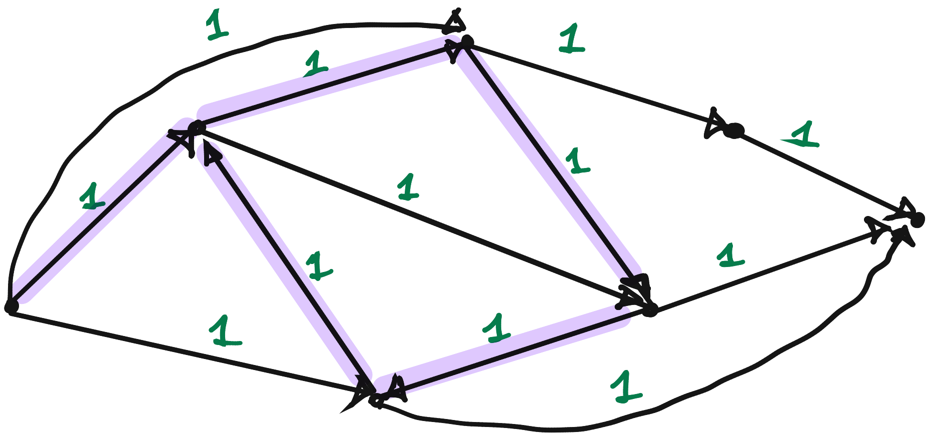
START WITH A FLOW OF VALUE K



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TRIVIALLY TRUE.

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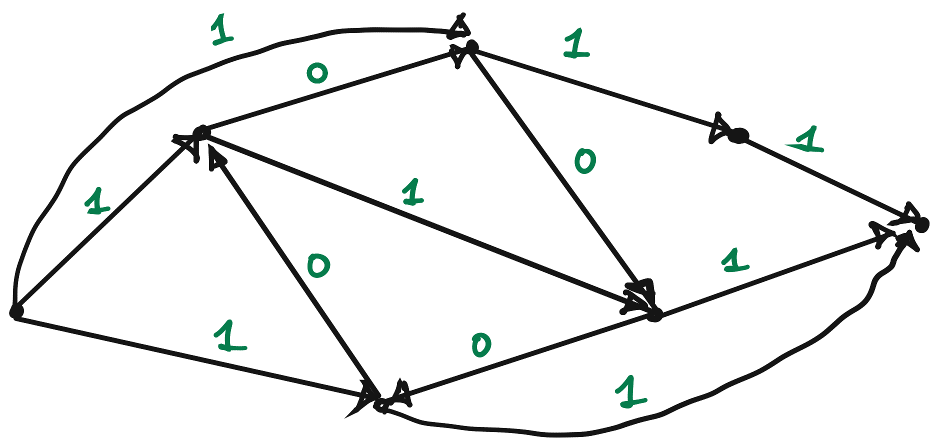
START WITH A FLOW OF VALUE K



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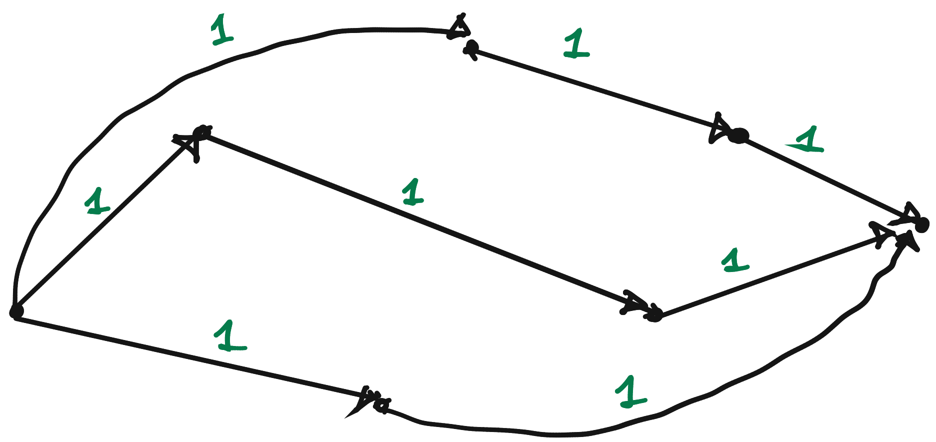
START WITH A FLOW OF VALUE K



FOUND A FLOW OF VALUE K THAT USES LESS EDGES.

RUNNING TIME :

RUNNING TIME :

$O(mn)$ FOR FF.

CIRCULATION WITH DEMAND

MAX FLOW : SINGLE SOURCE
SINGLE SINK

CIRCULATION WITH DEMAND

MAX FLOW : MULTIPLE SOURCE
MULTIPLE SINK

CIRCULATION WITH DEMAND

MAX FLOW : MULTIPLE SOURCE
MULTIPLE SINK

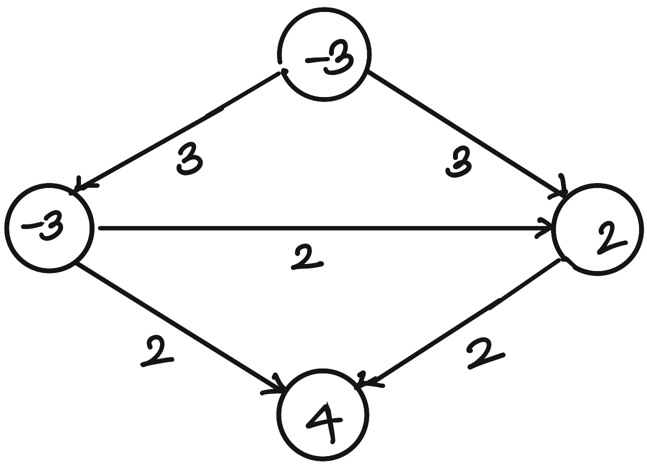
EACH SOURCE HAS A FIXED SUPPLY
& EACH SINK HAS A FIXED DEMAND

CIRCULATION WITH DEMAND

MAX FLOW : MULTIPLE SOURCE
MULTIPLE SINK

EACH SOURCE HAS A FIXED SUPPLY
& EACH SINK HAS A FIXED DEMAND

SATISFY ALL DEMANDS WITH AVAILABLE SUPPLY.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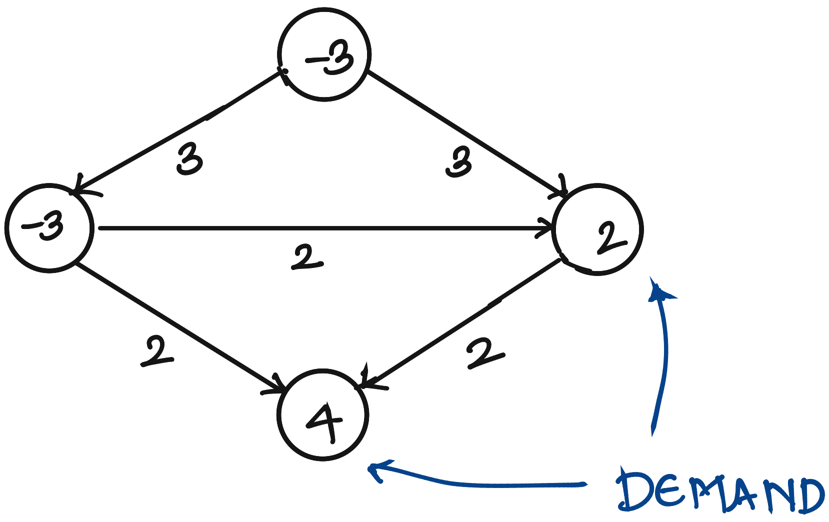

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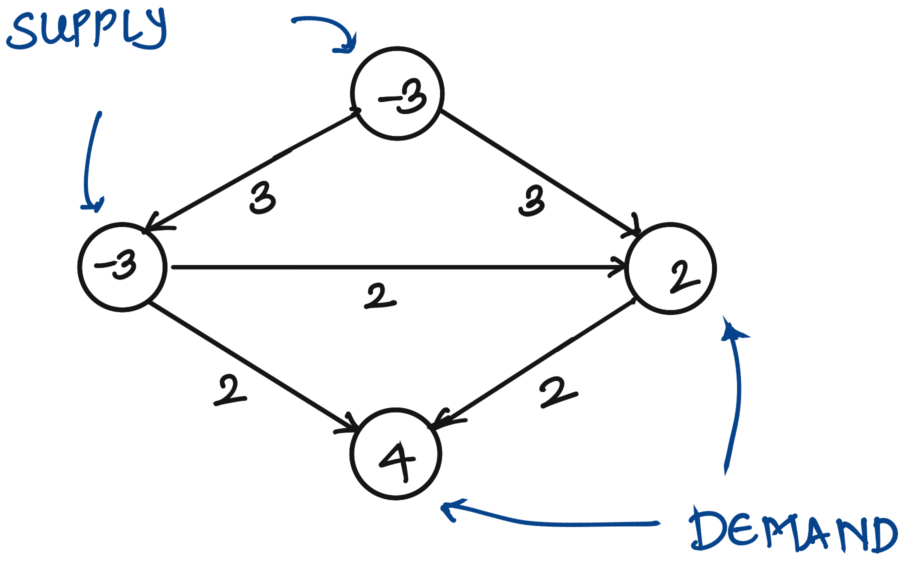
DEMAND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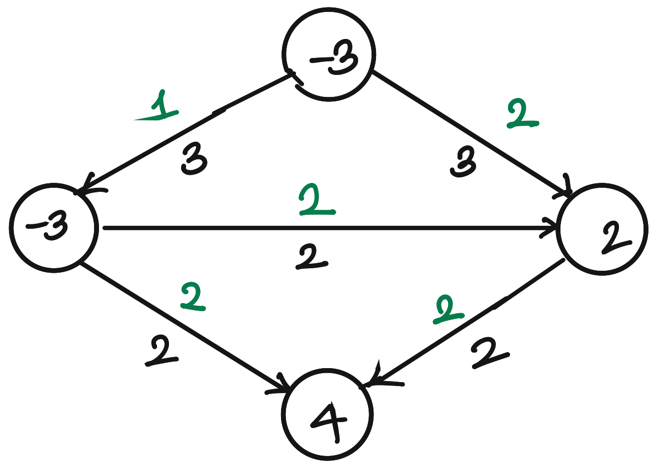
DEMAND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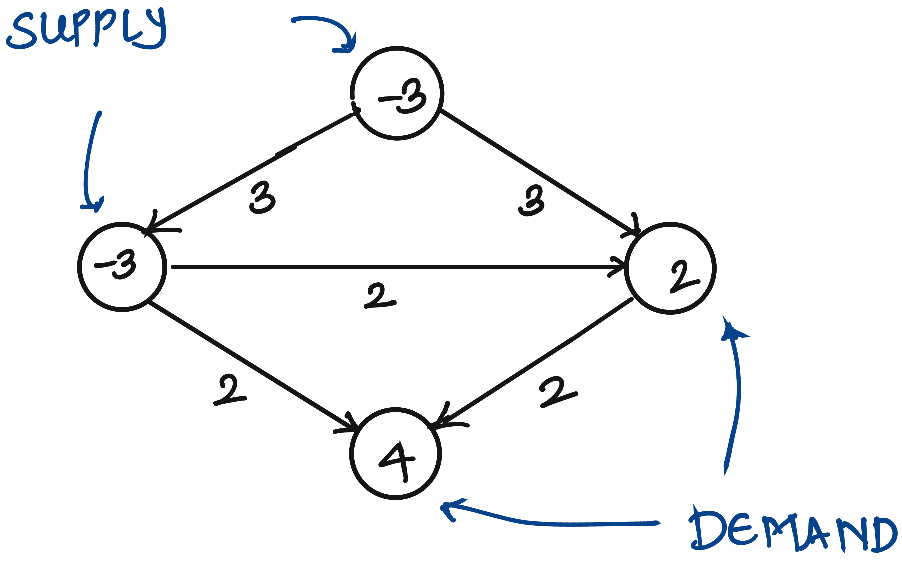
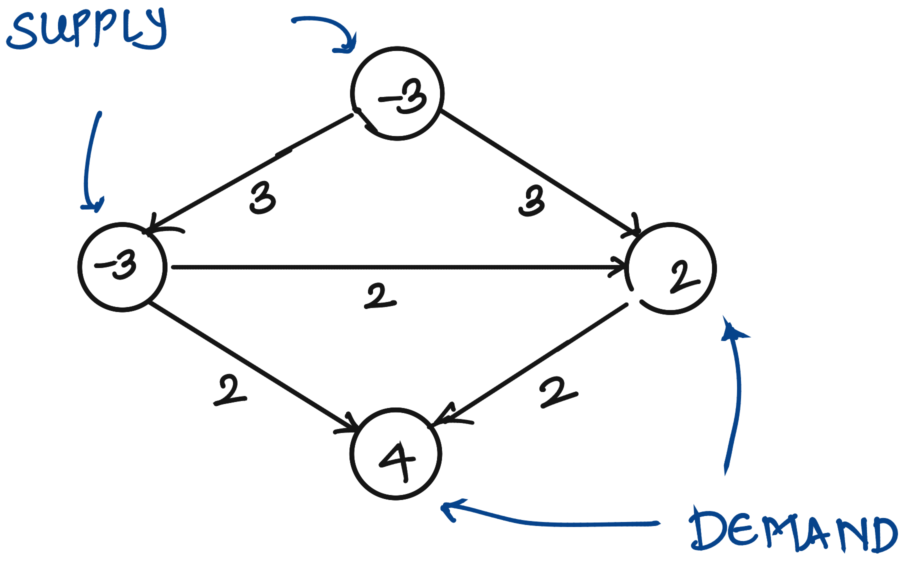


FOR EACH v HAS AN ASSOCIATED VALUE d_v

IF $d_v > 0$, THEN

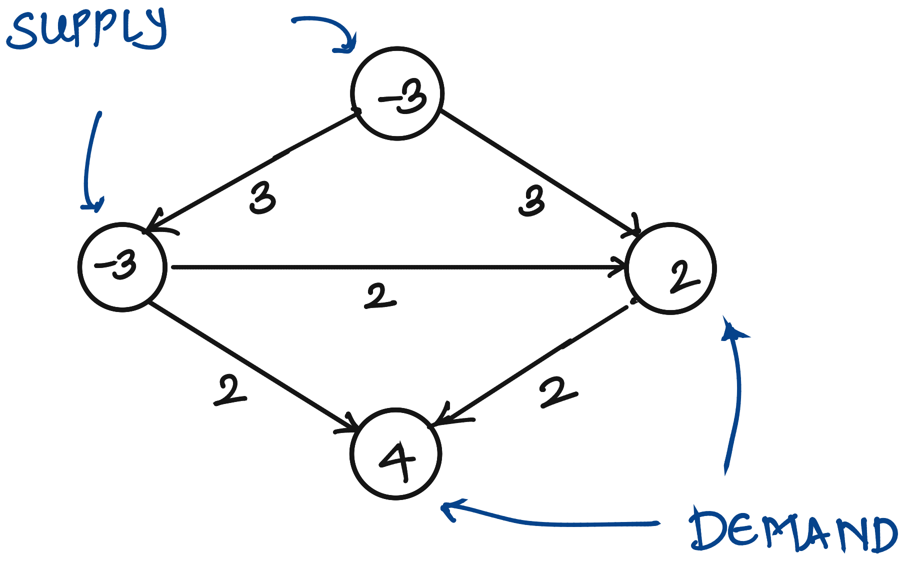


For each v has an associated value d_v

If $d_v > 0$, then its a demand

If $d_v < 0$, then its a supply

If $d_v = 0$,



FOR EACH v HAS AN ASSOCIATED VALUE d_v

If $d_v > 0$, THEN ITS A DEMAND

If $d_v < 0$, THEN ITS A SUPPLY

If $d_v = 0$, INTERNAL NODE

GIVEN A GRAPH G WITH CAPACITIES ON EACH EGDE, d_v FOR EACH NODE $v \in V$, ASSIGN A FLOW $f(e)$ ON EACH EDGE SUCH THAT

(i) $0 \leq f(e) \leq c(e)$

GIVEN A GRAPH G WITH CAPACITIES ON EACH EGDE, d_v FOR EACH NODE $v \in V$, ASSIGN A FLOW $f(e)$ ON EACH EDGE SUCH THAT

(i) $0 \leq f(e) \leq c(e)$

(ii) FOR EACH NODE v,

$$\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) - \sum_{e \text{ in } v} f(e) = -d_v$$

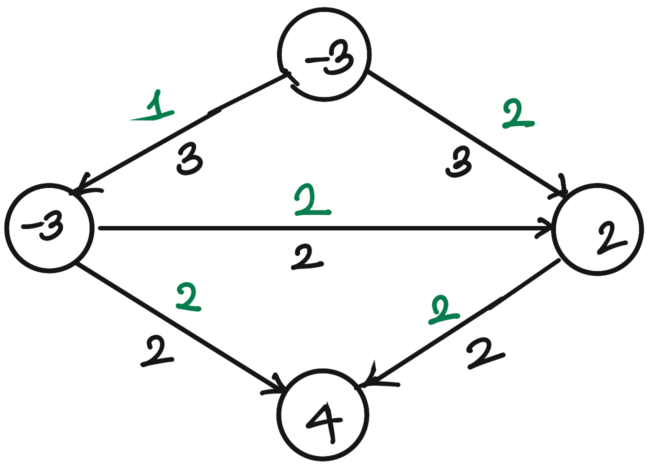
GIVEN A GRAPH G WITH CAPACITIES ON EACH EGDE, d_v FOR EACH NODE $v \in V$, ASSIGN A FLOW $f(e)$ ON EACH EDGE SUCH THAT

(i) $0 \leq f(e) \leq c(e)$

(ii) FOR EACH NODE v,

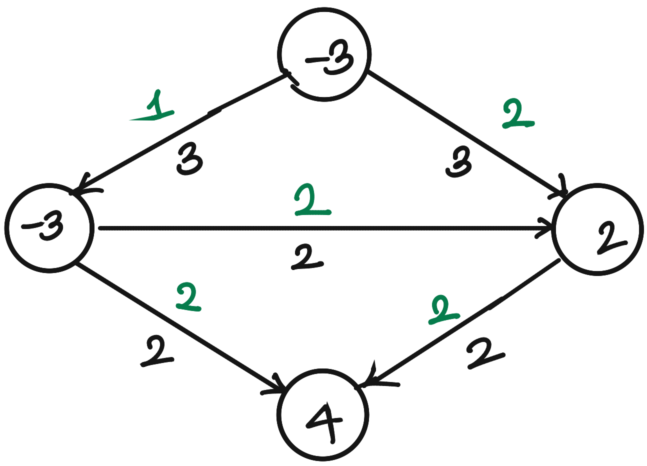
$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e) = -d_v$$

FEASABILITY PROBLEM: FIND IF THERE EXISTS A CIRCULATION IN G.



TOTAL SUPPLY = 6

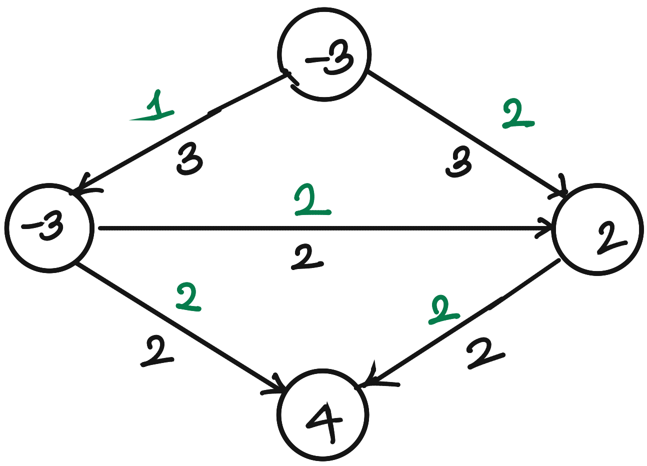
TOTAL DEMAND = 6



TOTAL SUPPLY = 6

TOTAL DEMAND = 6

<u>LEMMA</u>: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_{v} d_v = 0$



TOTAL SUPPLY = 6

TOTAL DEMAND = 6

LEMMA: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_v d_v = 0$

PROOF: IN A FEASIBLE CIRCULATION

$$\sum_{\substack{e \text{ OUT} \\ \text{of } v}} f(e) - \sum_{\substack{e \text{ IN} \\ v}} f(e) = -d_v$$

LEMMA: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_{v} d_v = 0$

PROOF: IN A FEASIBLE CIRCULATION

$$\sum_{\substack{e \text{ OUT} \\ \text{of } v}} f(e) - \sum_{\substack{e \text{ IN} \\ v}} f(e) = -d_v$$

$$\sum_{v \in V} \left(\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) - \sum_{\substack{e \text{ IN} \\ v}} f(e) \right) = -\sum_{v \in V} d_v$$

LEMMA: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_v d_v = 0$

PROOF: IN A FEASIBLE CIRCULATION

$$\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) - \sum_{\substack{e \text{ in} \\ v}} f(e) = -d_v$$

$$\sum_{v \in X} \left(\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) - \sum_{\substack{e \text{ in} \\ v}} f(e) \right) = -\sum_{v \in X} d_v$$

$$x \longrightarrow y$$

$+f(xy)$ by NODE x

$-f(xy)$ BY NODE y.

LEMMA: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_v d_v = 0$

PROOF: IN A FEASIBLE CIRCULATION

$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e) = -d_v$$

$$\sum_{v \in V} \left(\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e) \right) = -\sum_{v \in V} d_v$$

$$x \longrightarrow y$$

$+ f(xy)$ by NODE x

$- f(xy)$ BY NODE y.

$$0 = -\sum_{v \in V} d_v$$

$$\Rightarrow \sum_v d_v = 0$$

LEMMA: IF THERE EXISTS A FEASIBLE CIRCULATION, THEN $\sum_v d_v = 0$

PROOF: IN A FEASIBLE CIRCULATION

$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e) = -d_v$$

$$\sum_{v \in V} \left(\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e) \right) = -\sum_{v \in V} d_v$$

$$x \longrightarrow y$$

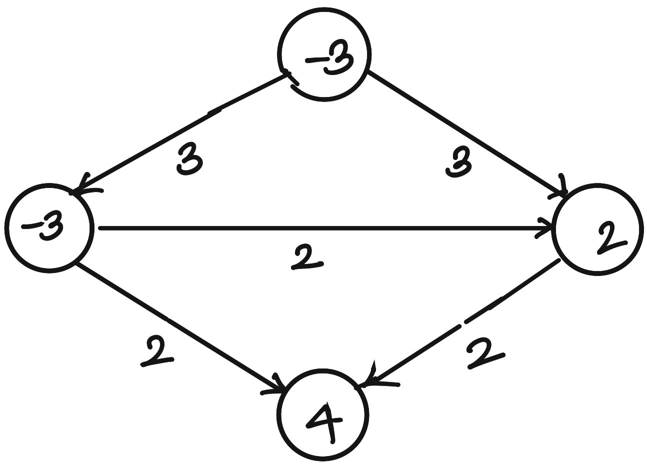
$+f(xy)$ by NODE x

$-f(xy)$ BY NODE y.

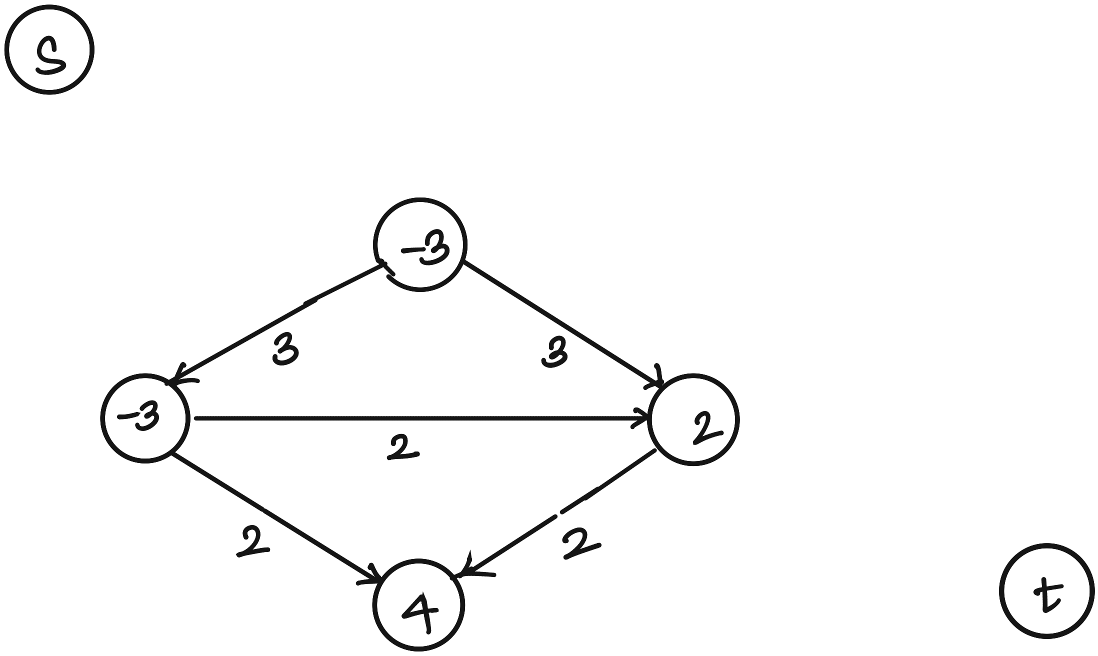
$$0 = -\sum_{v \in V} d_v$$

$$\Rightarrow \sum_v d_v = 0$$

$$\Rightarrow \sum_{v : d_v < 0} -d_v = \sum_{v : d_v > 0} d_v$$



CONVERT THE CIRCULATION PROBLEM INTO A FLOW PROBLEM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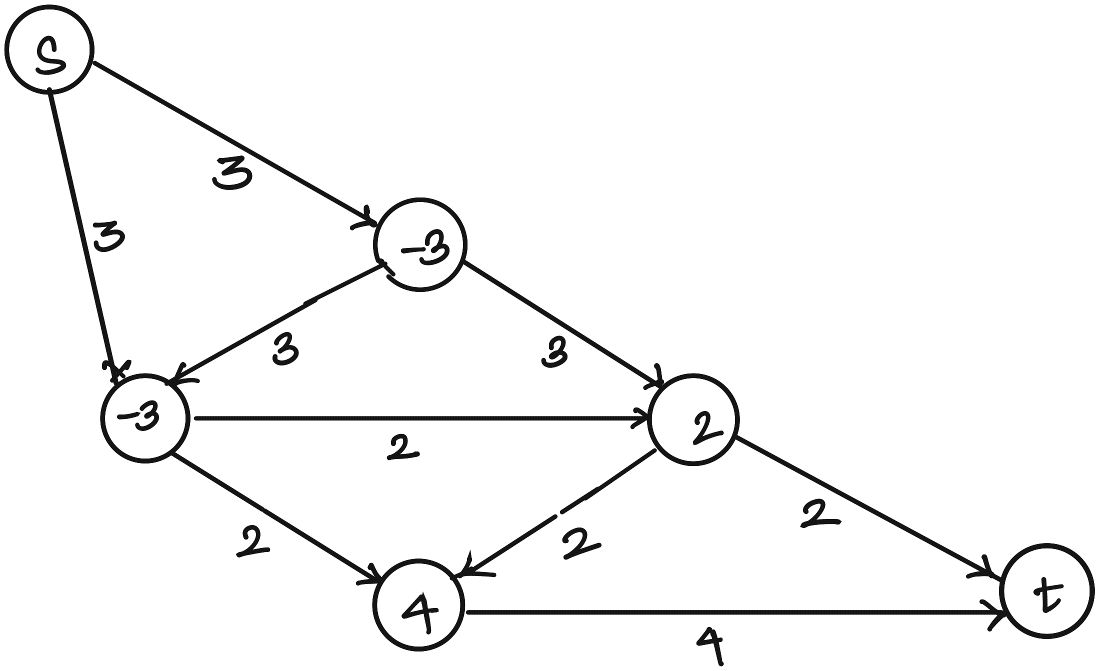
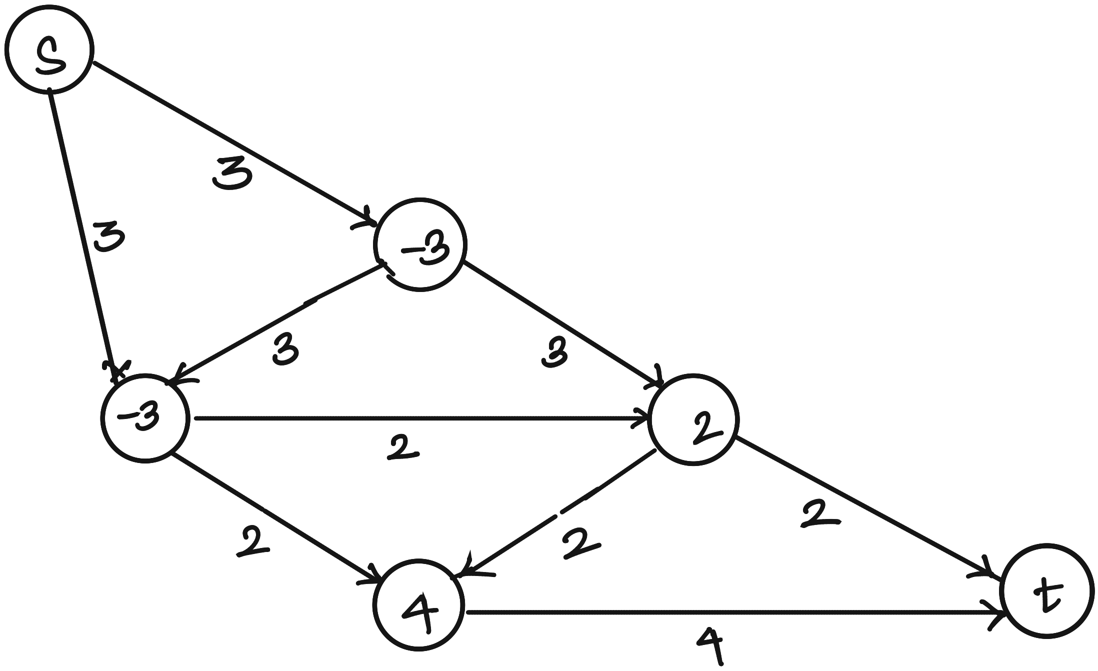


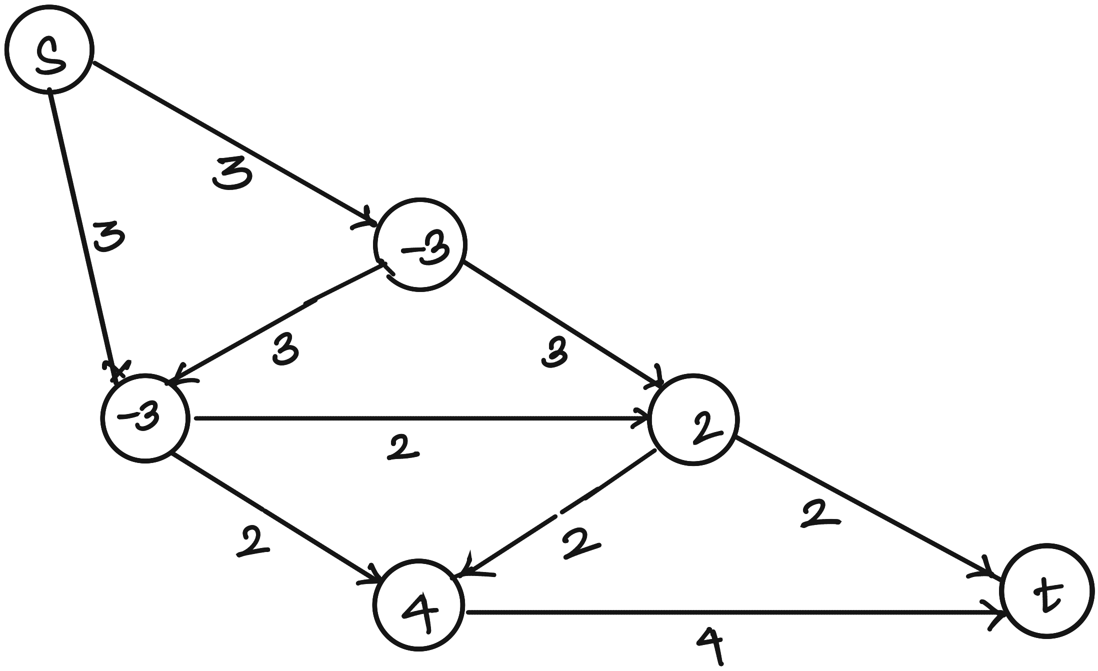
FOR EACH VERTEX v WITH $d_v < 0$,
ADD $s \xrightarrow[d_v]{} v$

FOR EACH VERTEX v WITH $d_v > 0$

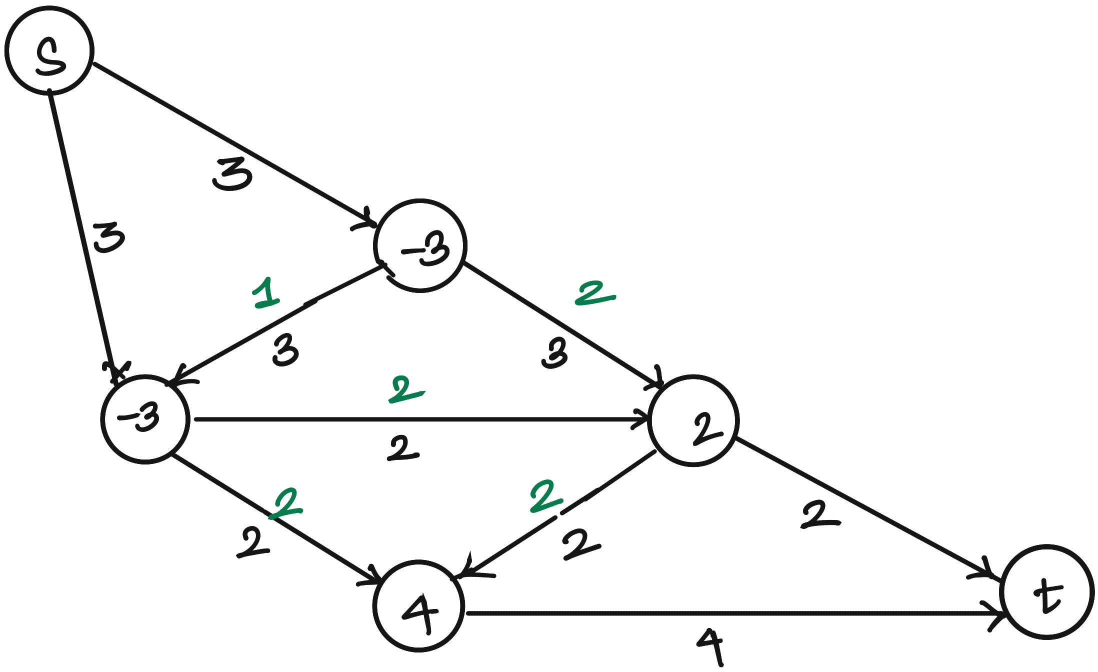
ADD $v \xrightarrow[d_v]{} t$



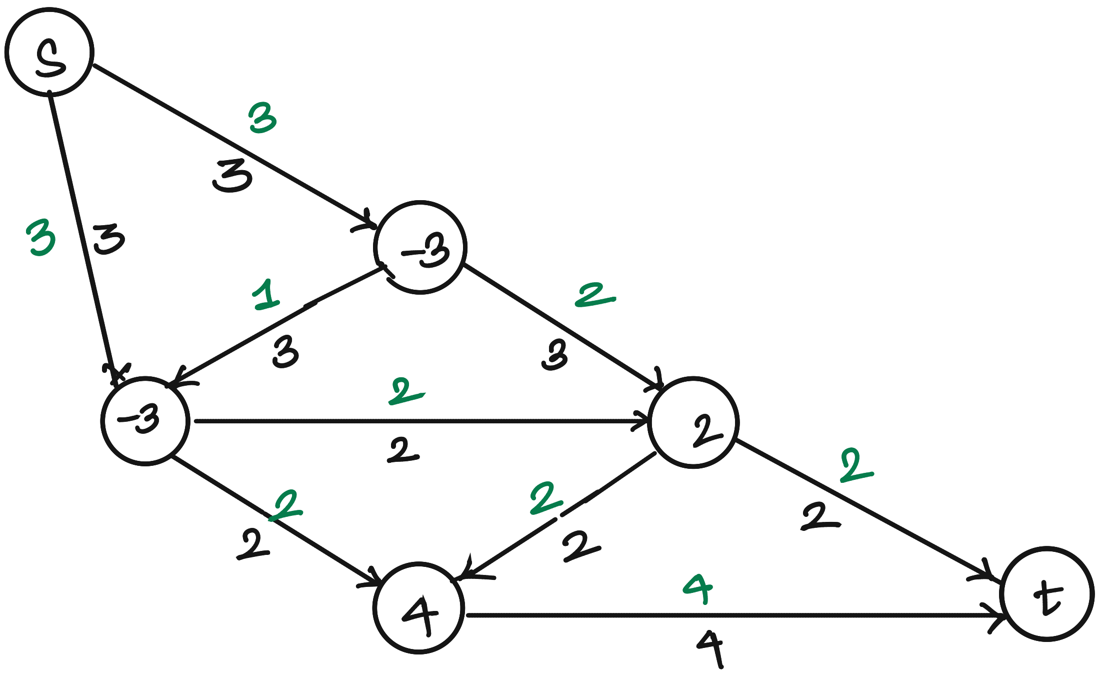
<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE ___ IN G'



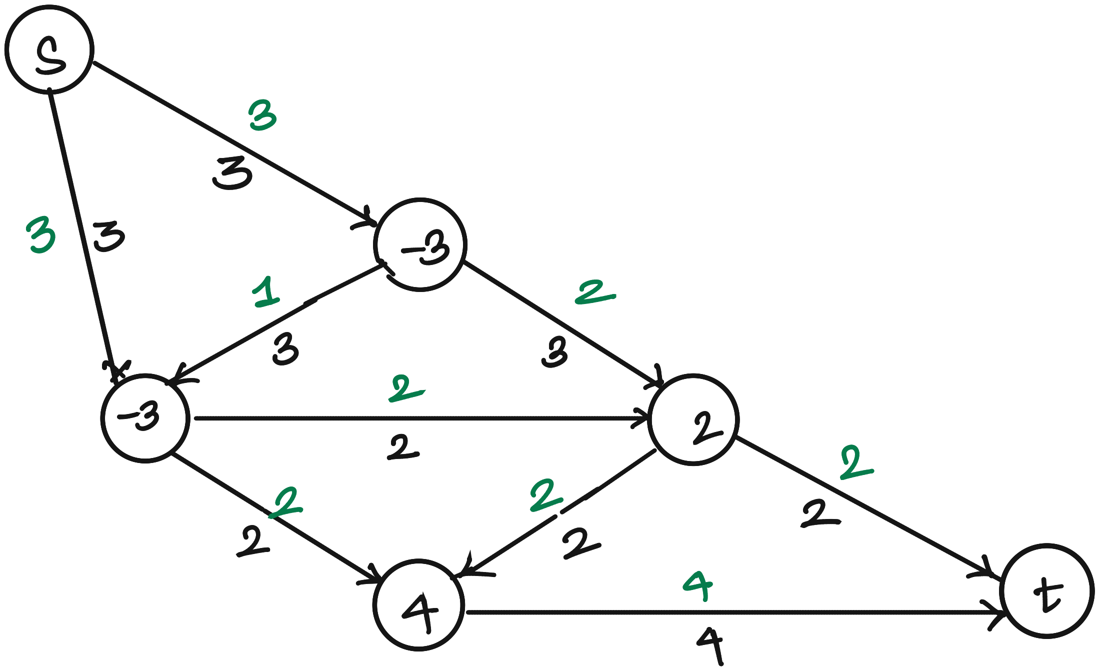
<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v : d_v < 0} -d_v$ IN G'



<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} -d_v$ IN G'



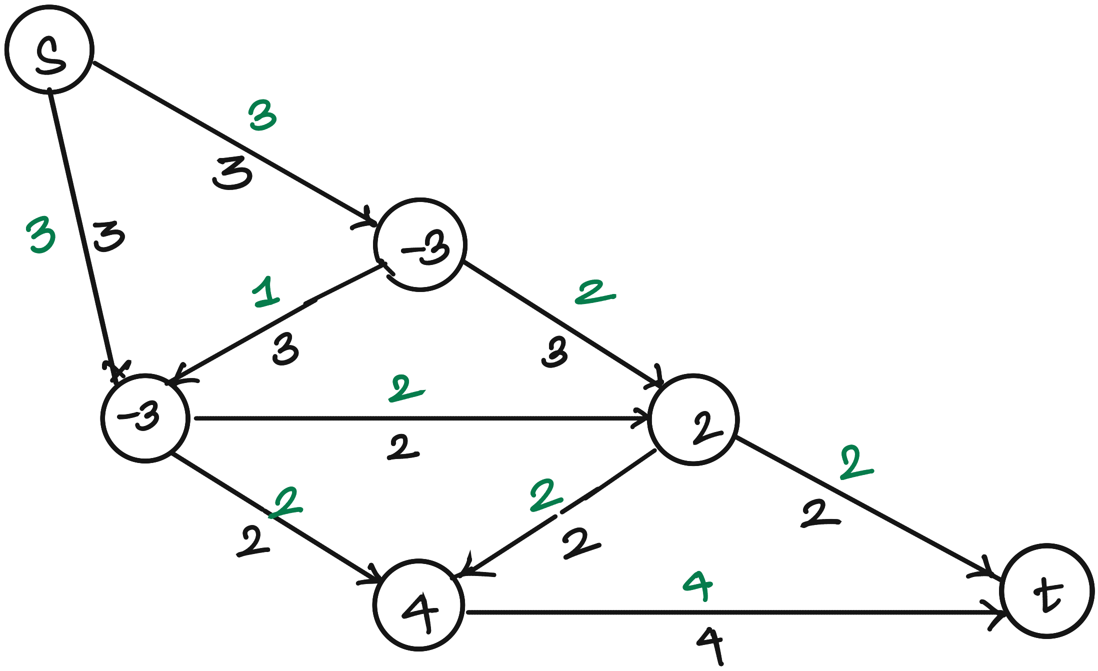
LEMMA: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} -d_v$ IN G'



<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v : d_v < 0} -d_v$ IN G'

FOR EVERY NODE v IN G WITH $d_v < 0$

$$\sum_{\substack{e \text{ out} \\ \text{of } v \\ \text{in } G}} f(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f(e) = -d_v$$

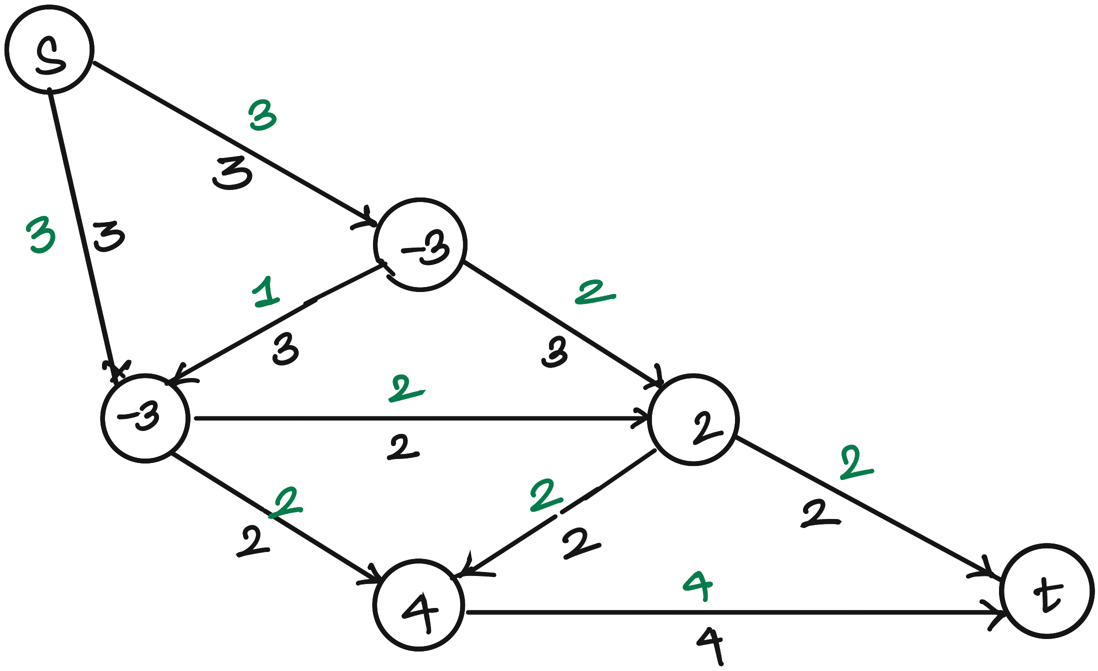


<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} d_v$ IN G'

FOR EVERY NODE v IN G WITH $d_v < 0$

$$\sum_{\substack{e \text{ out of } v \\ \text{in } G}} f(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f(e) = -d_v$$

$$\sum_{\substack{e \text{ out of } v \\ \text{in } G}} f(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f(e) + d_v = 0$$

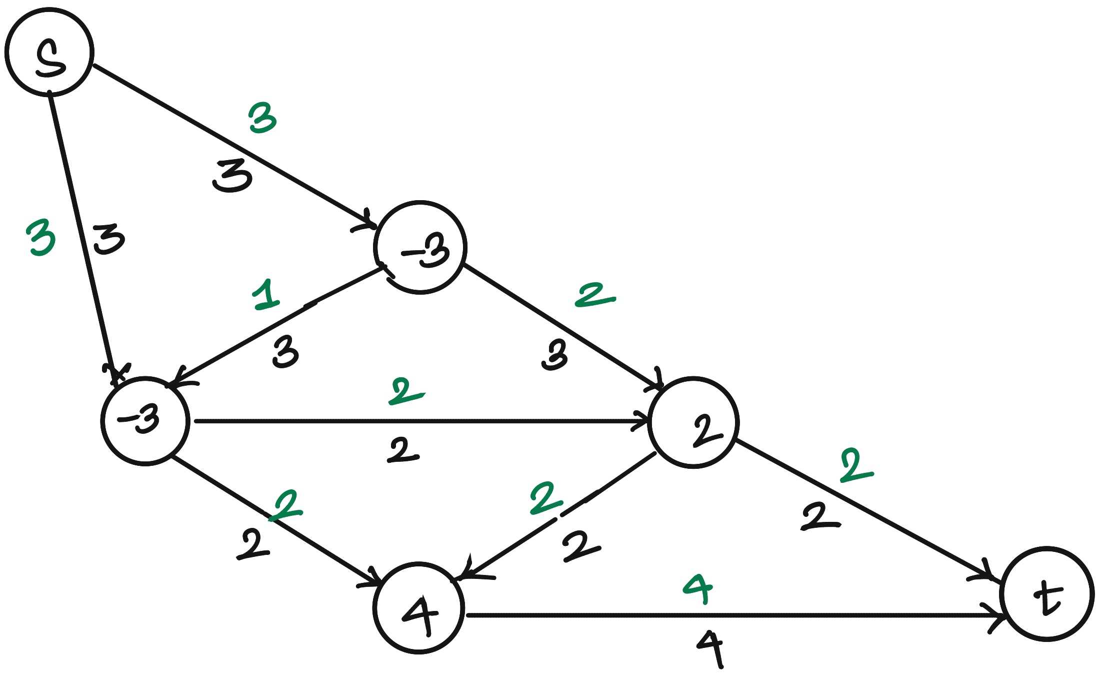


LEMMA: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} d_v$ IN G'

FOR EVERY NODE v IN G WITH $d_v < 0$

$$\sum_{\substack{e \text{ out} \\ \text{of } v \\ \text{in } G}} f(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f(e) = -d_v$$

$$\sum_{\substack{e \text{ out} \\ \text{of } v \\ \text{in } G}} f(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f(e) + f(sv) = 0$$



<u>LEMMA</u>: IF THERE IS A FEASIBLE CIRCULATION IN G, THEN THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} d_v$ IN G'

FOR EVERY NODE v IN G WITH $d_v < 0$

$$\sum_{e \text{ out of } v \text{ in } G} f(e) - \sum_{e \text{ in } v \text{ in } G} f(e) = -d_v$$

$$\sum_{e \text{ out of } v \text{ in } G'} f(e) - \sum_{e \text{ in } v \text{ in } G'} f(e) = 0$$

$\Leftarrow$ LEMMA: IF THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} -d_v$ IN G', THEN THERE IS A FEASIBLE CIRCULATION IN G.

$\Leftarrow$ LEMMA: IF THERE IS A FLOW OF VALUE $\sum_{v : d_v < 0} -d_v$ IN G', THEN THERE IS A FEASIBLE CIRCULATION IN G.

RUNNING TIME:

$\Leftarrow$ LEMMA: IF THERE IS A FLOW OF VALUE $\sum_{v: d_v < 0} -d_v$ IN G', THEN THERE IS A FEASIBLE CIRCULATION IN G.

RUNNING TIME: $O\left(m\left(\sum_{v: d_v < 0} -d_v\right)\right)$

GIVEN A GRAPH G WITH CAPACITIES ON EACH EGDE, d_v FOR EACH NODE $v \in V$, ASSIGN A FLOW $f(e)$ ON EACH EDGE SUCH THAT

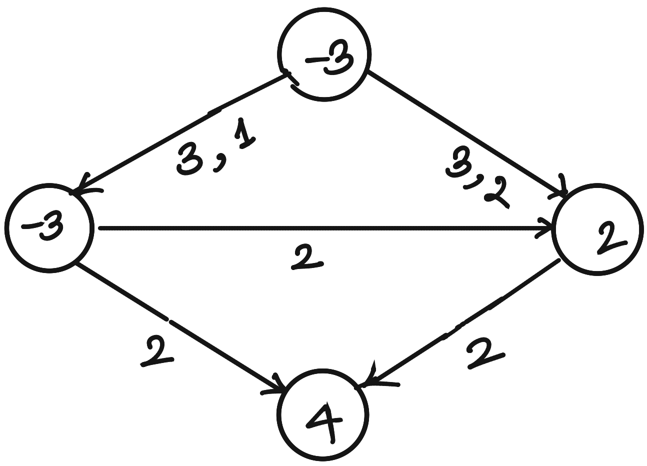
(i) $\ell(e) \leq f(e) \leq c(e)$

(ii) FOR EACH NODE v,

$$\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) - \sum_{e \text{ in } v} f(e) = -d_v$$

FEASABILITY PROBLEM: FIND IF EXISTS A CIRCULATION IN G.

CIRCULATION WITH DEMANDS & LOWER BOUND.



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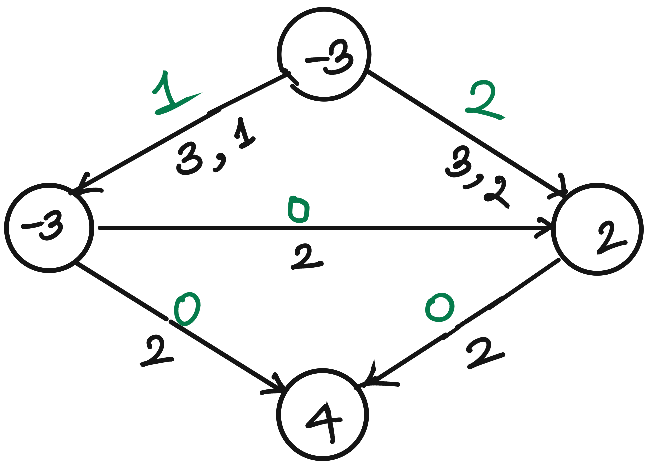
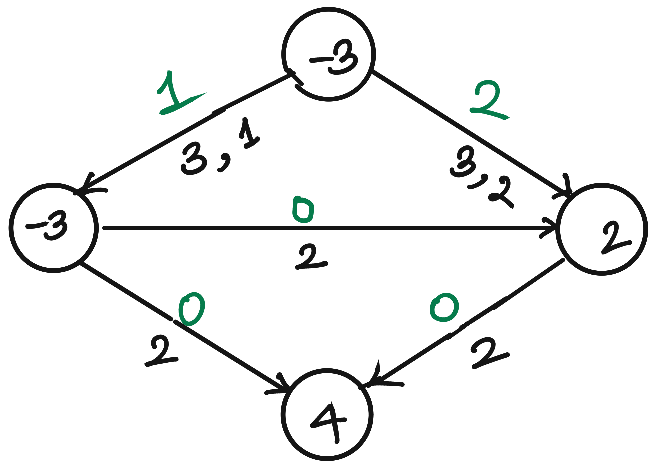


INITIALLY $f(e) = \ell(e)$



INITIALLY $f(e) = l(e)$

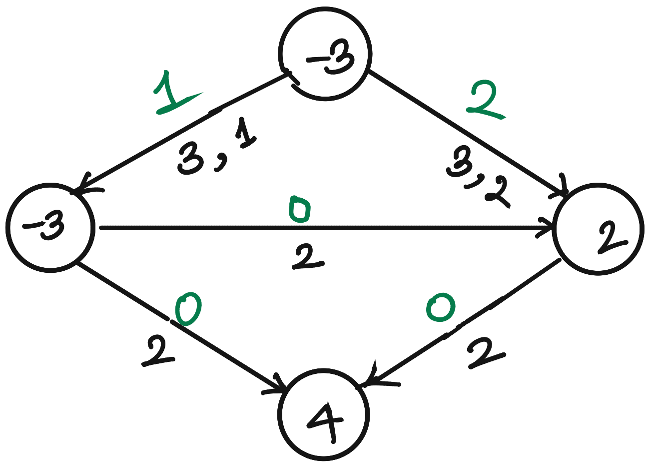
MAY OR MAY NOT BE A CIRCULATION.

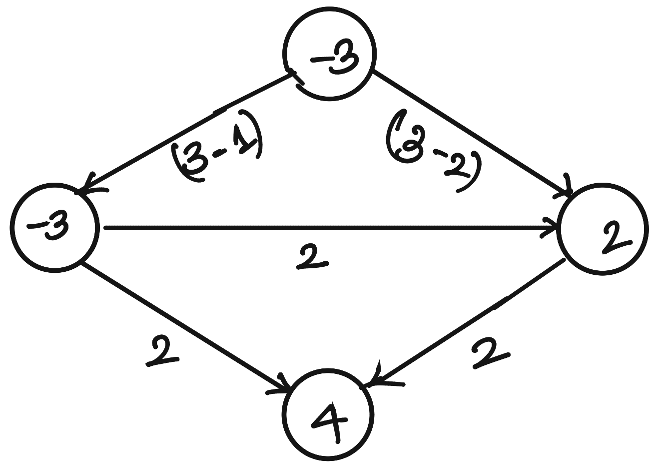
$\Rightarrow$ CONVERT CIRCULATION WITH DEMAND

f LOWER BOUND

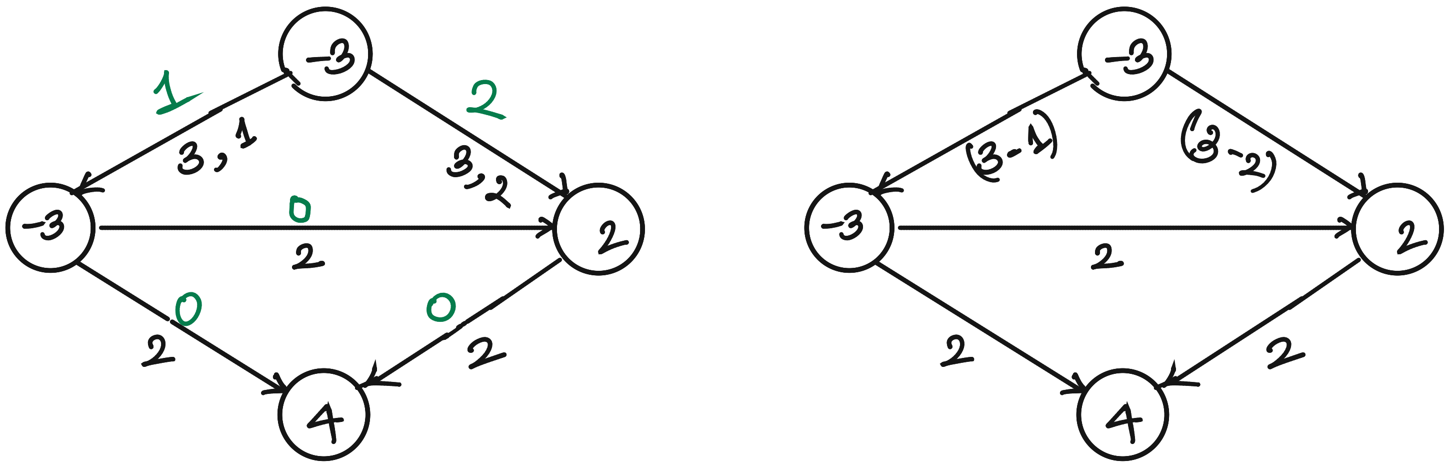
$\Downarrow$

CIRCULATION WITH DEMAND



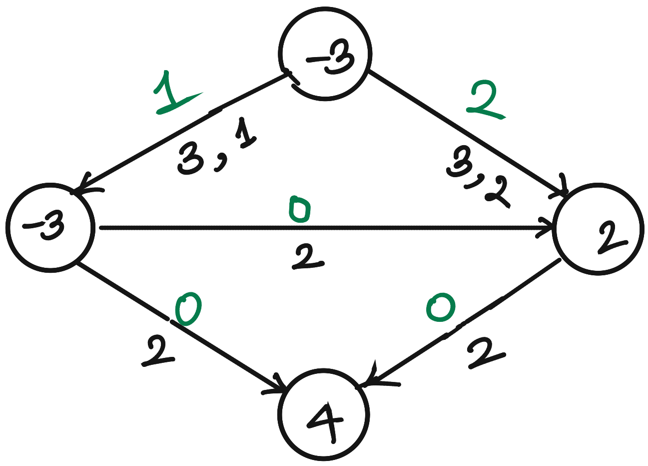


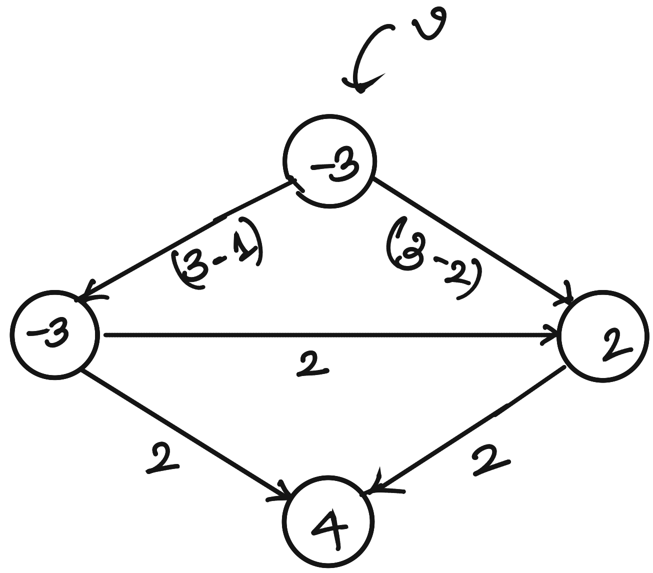
JUST NEED TO CALCULATE THE ADDITIONAL FLOW ON EACH EDGE.



JUST NEED TO CALCULATE THE ADDITIONAL FLOW ON EACH EDGE.

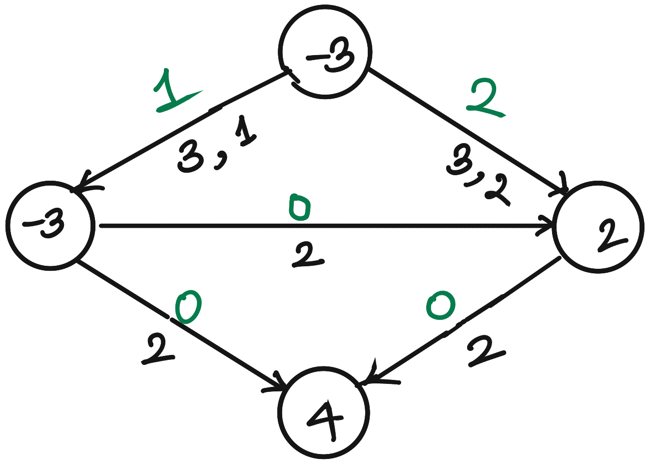
Q: BUT WHAT ABOUT THE dv? SHOULD THEY ALSO CHANGE?

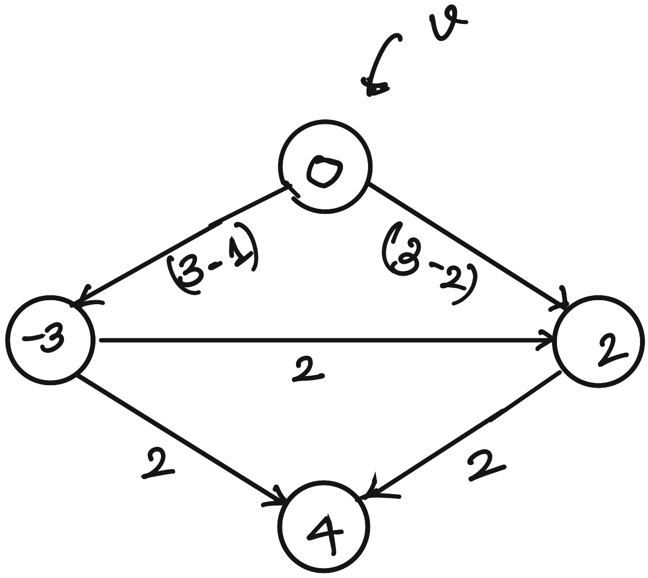




$$\sum_{e \text{ out of } v} f_1(e) - \sum_{e \text{ in } v} f_1(e)$$

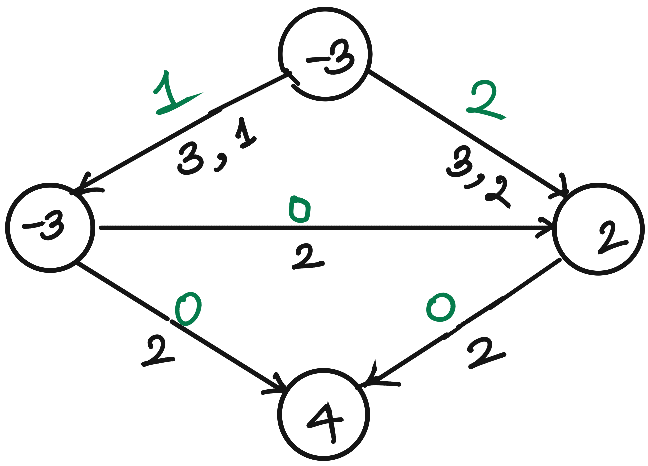
$$= 1 + 2 = 3$$

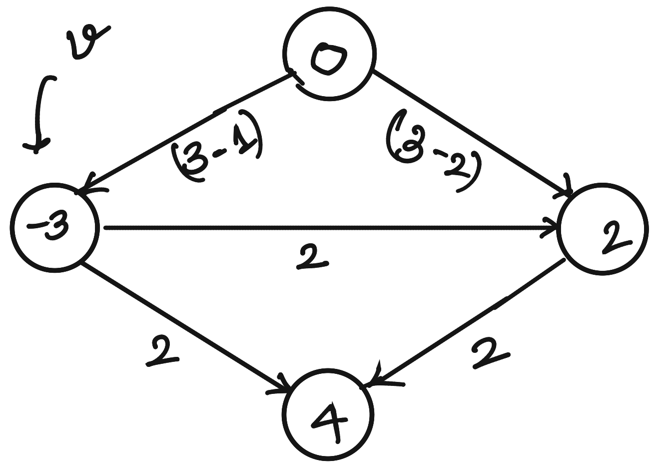




$$\sum_{e \text{ out of } v} f_1(e) - \sum_{e \text{ in } v} f_1(e)$$

$$= 1 + 2 = 3 = -d_v$$



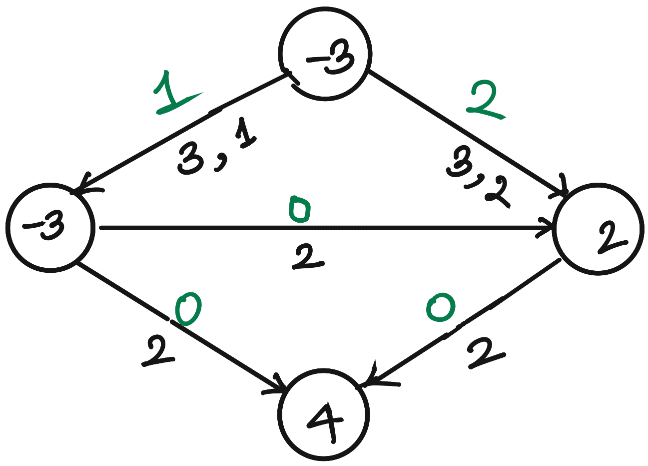


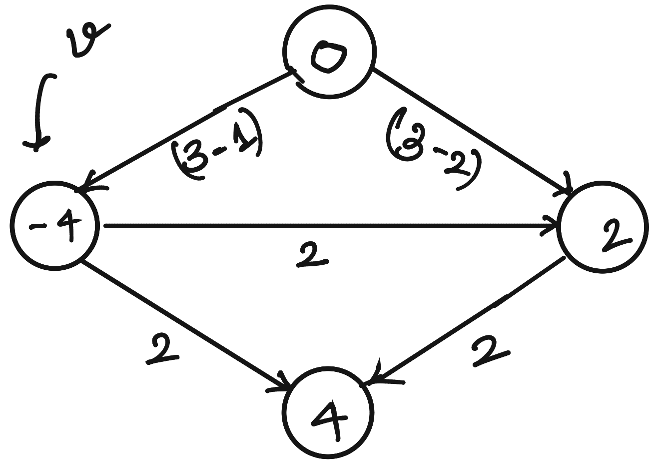
$$\sum_{e \text{ out of } v} f_1(e) - \sum_{e \text{ in } v} f_1(e)$$

$$= 0 - 1 = -1$$

IMBALANCE IN FLOW AT v =

=



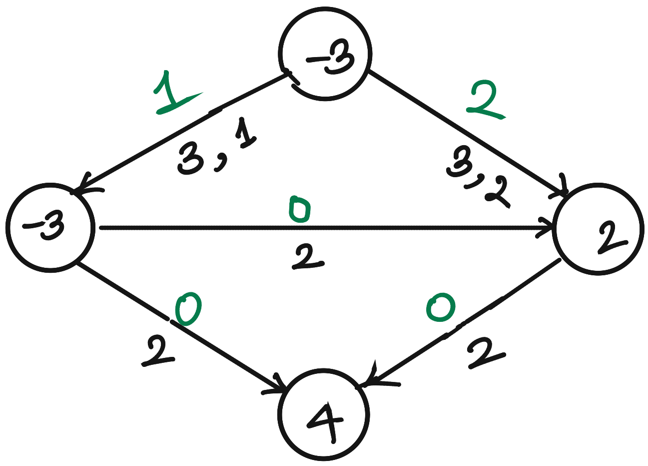


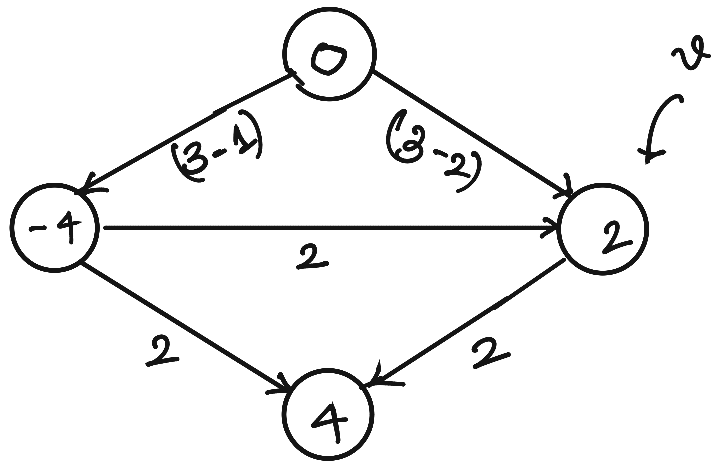
$$\sum_{\substack{e \text{ out} \\ \text{of } v}} f(e) \; - \; \sum_{\substack{e \text{ in} \\ v}} f(e)$$

$$= 0 - 1 = -1$$

IMBALANCE IN FLOW AT $v = -(-3) - (-1)$

$= 4$

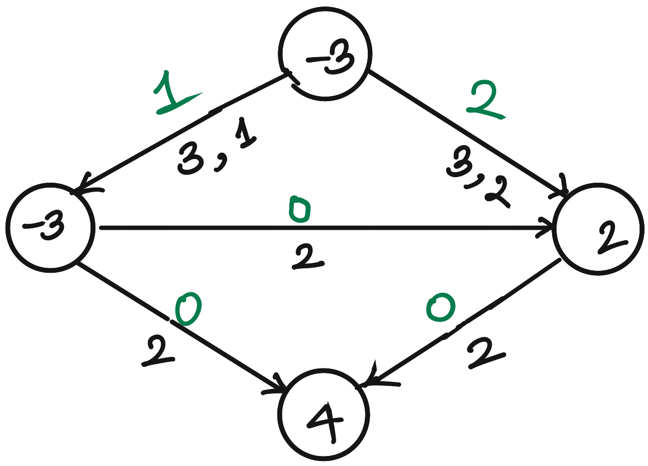


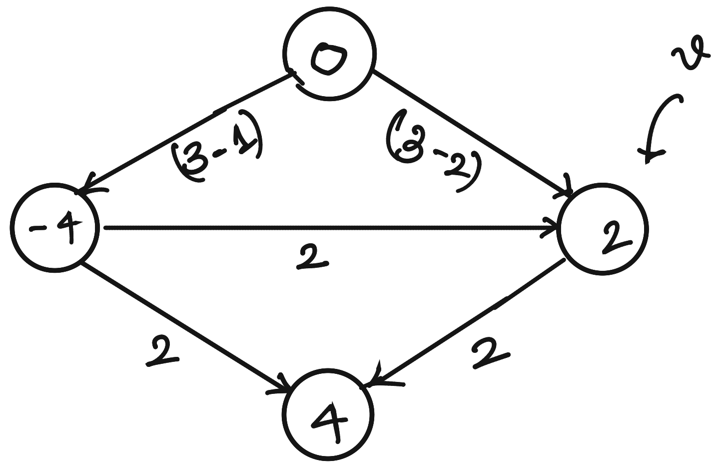


$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e)$$

$$0 - 2 = -2$$

IMBALANCE IN FLOW AT v =

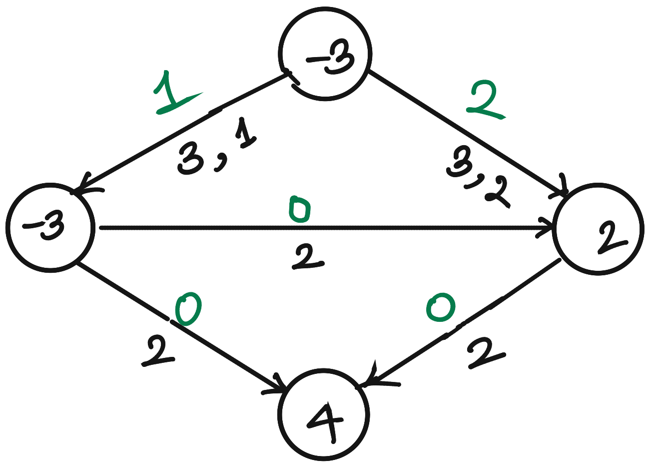


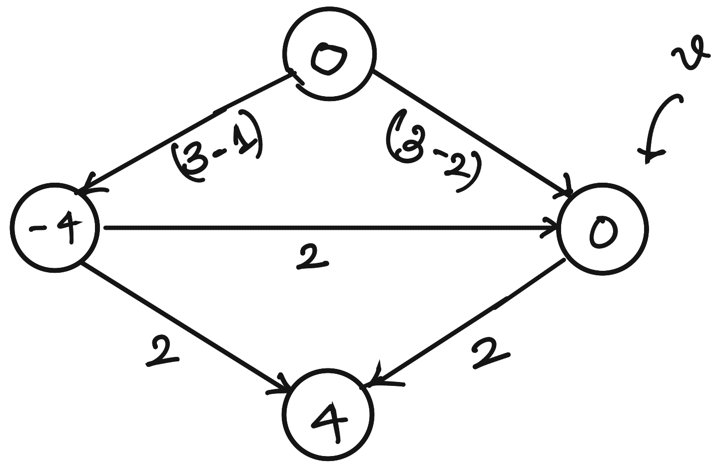


$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e)$$

$$0 - 2 = -2$$

IMBALANCE IN FLOW AT v $= -2-(-2)$
$= 0$



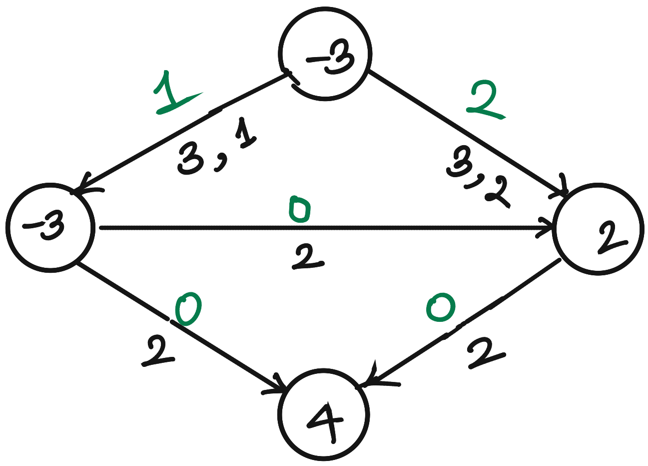


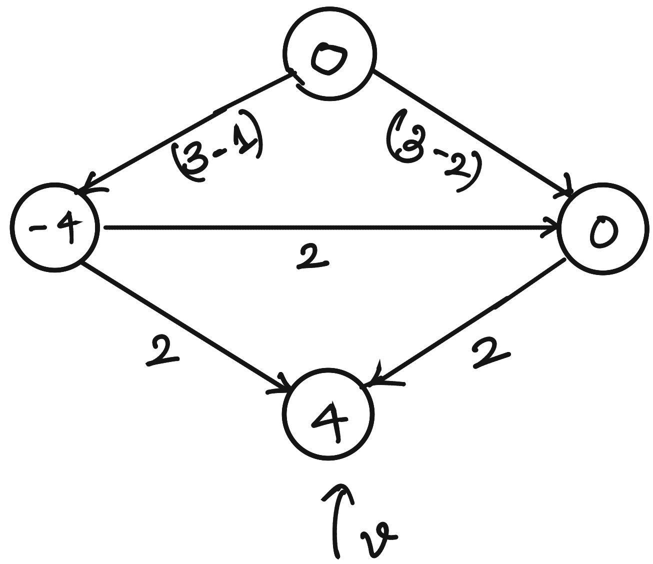
$$\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ in } v} f(e)$$

$$0 - 2 = -2$$

Imbalance in flow at $v = -2-(-2)$

$= 0$

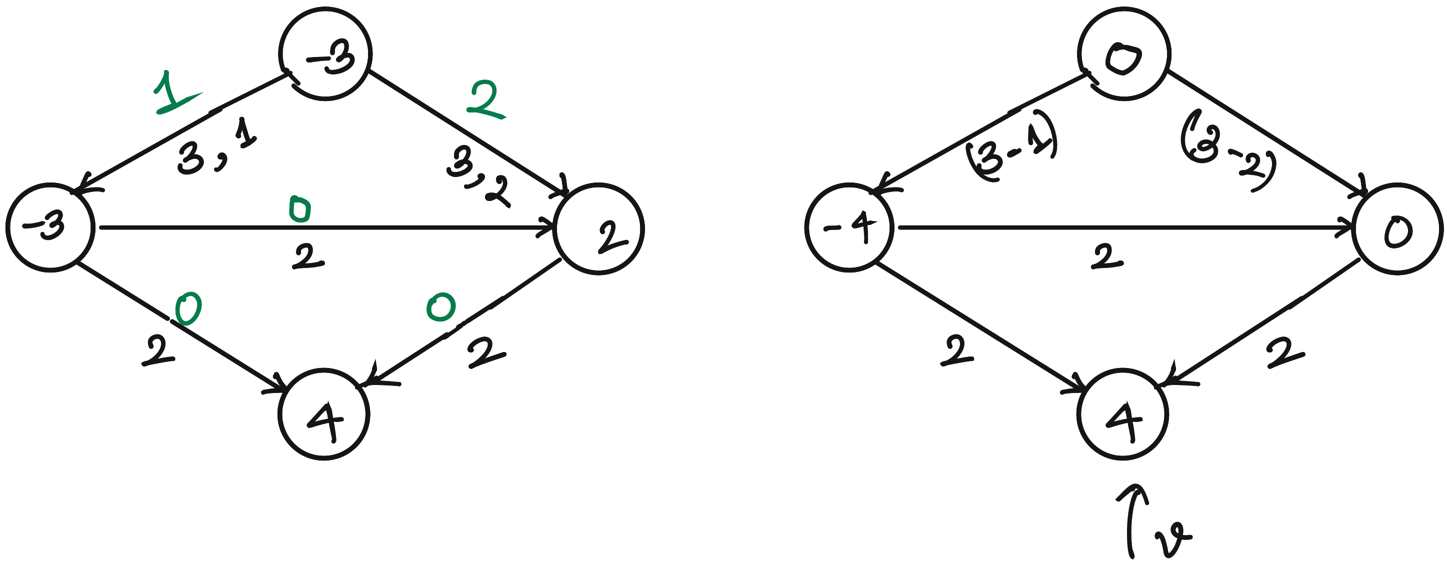




$$\sum_{e \text{ out of } v} f(e) \quad - \quad \sum_{e \text{ in } v} f(e)$$

$$0 \quad - \quad 0 = 0$$

IMBALANCE IN FLOW AT v =

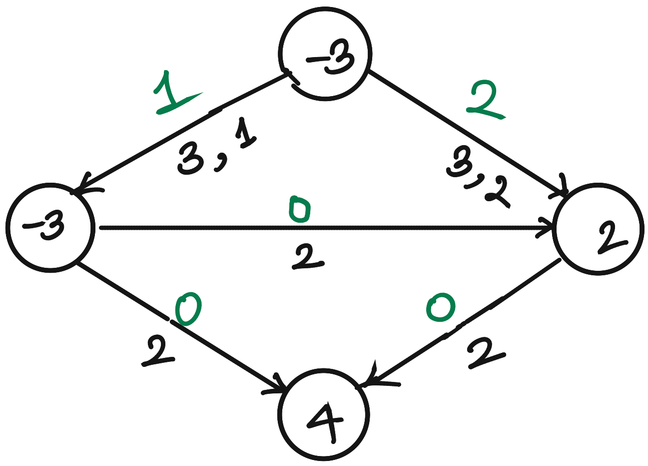


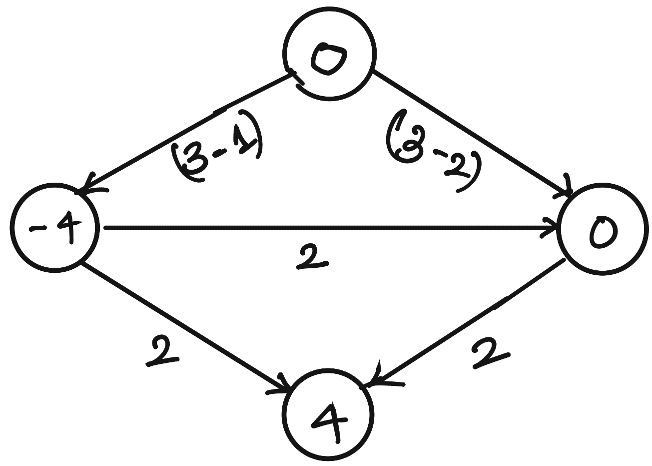
$$\sum_{e \text{ out of } v} f(e) \; - \; \sum_{e \text{ in } v} f(e)$$

$$0 \; - \; 0 = 0$$

IMBALANCE IN FLOW AT $v = -4 - 0$

$= -4$





G'

FIND IF THERE EXISTS A CIRCULATION IN G'.

$$\sum_{e \text{ out of } v} f'(e) - \sum_{e \text{ in } v} f'(e)$$

$$0 - 0 = 0$$

IMBALANCE IN FLOW AT $v = -4 - 0$

$= -4$

- → FIND THE INITIAL FLOW f_1.
- → MAKE A NEW GRAPH G' ON SAME NODES AS G.
- → $C'(e) =$

→ FIND THE INITIAL FLOW f_1.

→ MAKE A NEW GRAPH G' ON SAME NODES AS G.

→ $C'(e) = f(e) - l(e)$

→ FOR each v IN G',

LET $L_v = \sum_{\substack{e \text{ OUT} \\ \text{of } v \\ \text{in } G}} f_1(e) - \sum_{\substack{e \text{ IN} \\ v \\ \text{in } G}} f_1(e)$

$d'_v =$

→ FIND THE INITIAL FLOW f_1.

→ MAKE A NEW GRAPH G' ON SAME NODES AS G.

→ $c'(e) = f(e) - l(e)$

→ FOR each v IN G',

LET $L_v = \sum_{\substack{e \text{ OUT} \\ \text{of } v \\ \text{in } G}} f_1(e) - \sum_{\substack{e \text{ IN} \\ v \\ \text{in } G}} f_1(e)$

$$\boxed{d'_v = d_v + L_v}$$

IMBALANCE AT v $= -d_v - L_v$

$= -(d_v + L_v)$

→ FIND THE INITIAL FLOW f_1.

→ MAKE A NEW GRAPH G' ON SAME NODES AS G.

→ $c'(e) = f(e) - l(e)$

→ FOR each v IN G',

LET $L_v = \sum_{\substack{e \text{ out of } v \\ \text{in } G}} f_1(e) - \sum_{\substack{e \text{ in } v \\ \text{in } G}} f_1(e)$

$$\boxed{d'_v = d_v + L_v}$$

IMBALANCE AT v $= -d_v - L_v$

$= -(d_v + L_v)$

$\Uparrow$

NEW GRAPH G'

FIND IF THERE EXISTS A CIRCULATION IN G'.

PROBLEM: COMPANY : SELLS K PRODUCTS
CONDUCT A SURVEY OF n CUSTOMERS
TO FIND THEIR PREFERENCE.

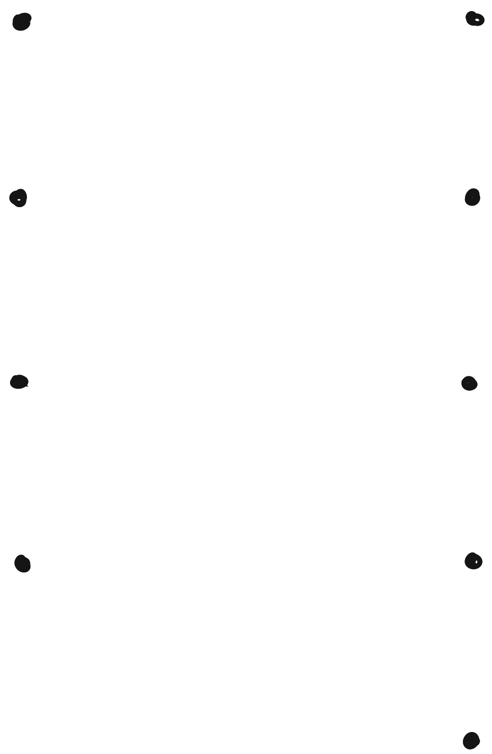
SURVEY

- EACH CUSTOMER RECEIVES A SUBSET OF K PRODUCTS
- A CUSTOMER CAN ONLY BE ASKED ABOUT PRODUCTS SHE HAS PURCHASED
- SHOULD ASK ATMOST $[c_i, c_i']$ QUESTIONS TO CUSTOMER i
- GIVEN A PRODUCT j, $[p_j, p_j']$ CUSTOMERS SHOULD HAVE BEEN ASKED QUESTIONS ABOUT PRODUCT j.

$G(A, B)$

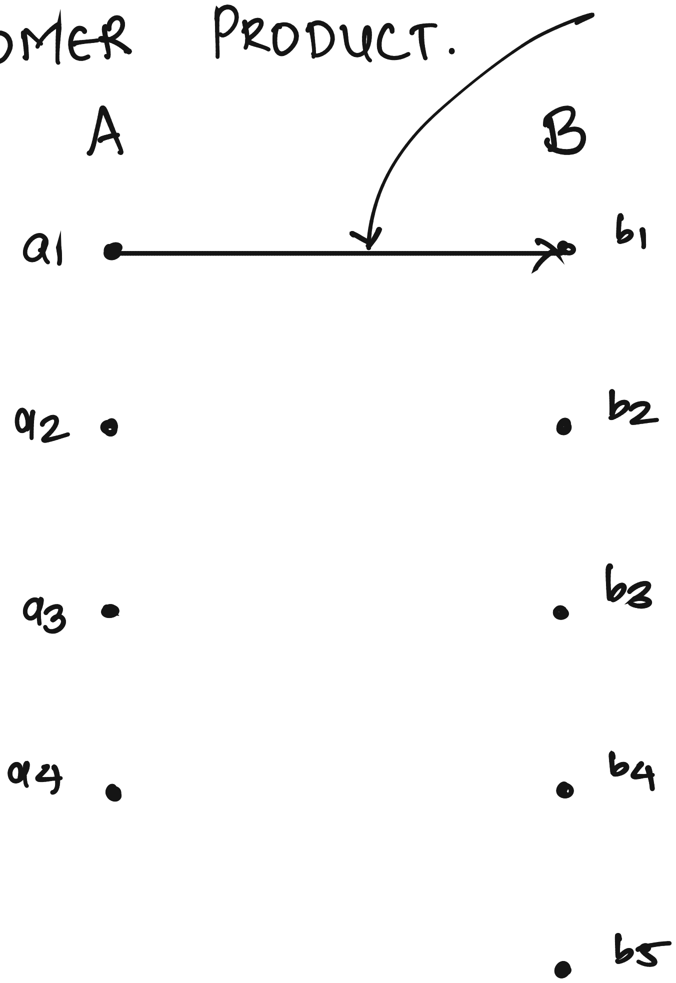
CUSTOMER PRODUCT.

A B



$G(A, B)$

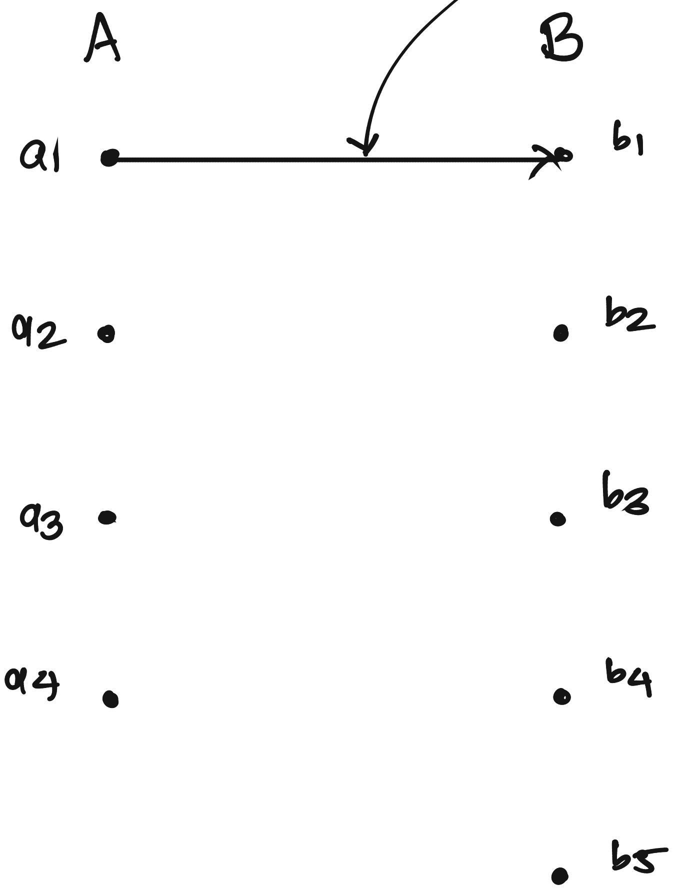
CUSTOMER PRODUCT.



$G(A, B)$

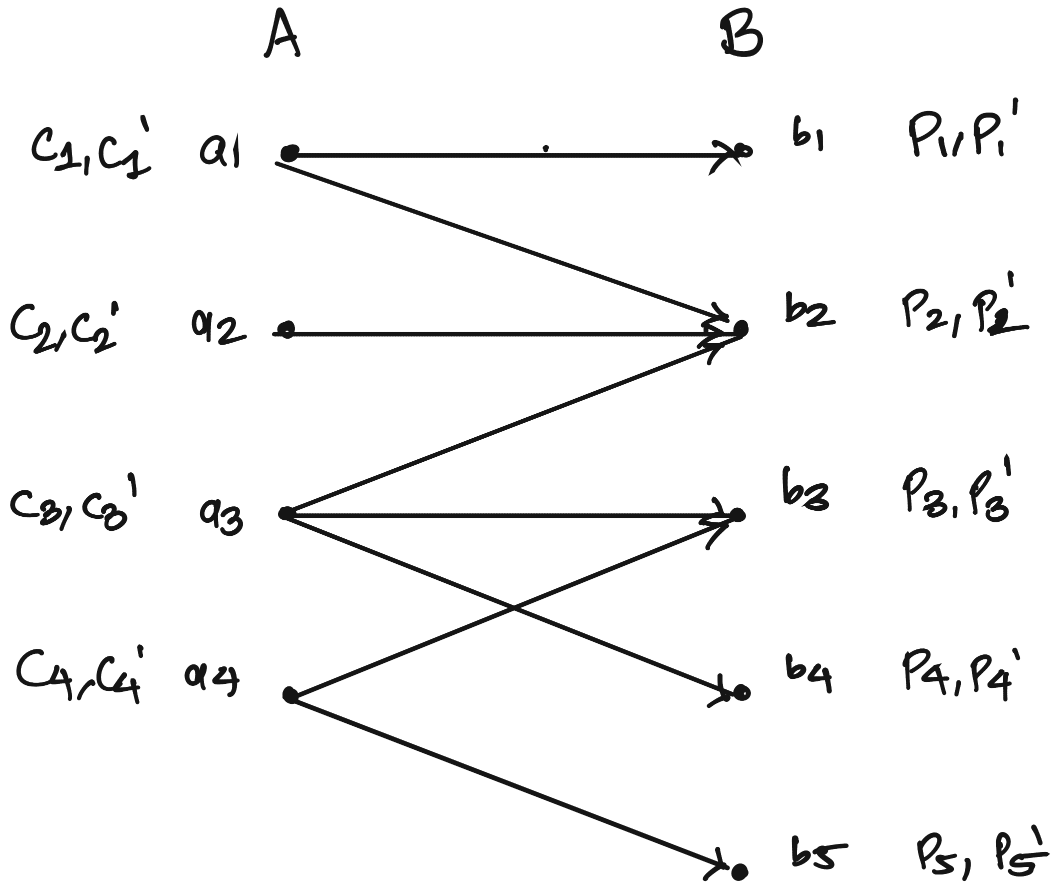
CUSTOMER PRODUCT.

CUSTOMER a_1 HAS BOUGHT PRODUCT b_1 $\Leftrightarrow$ (a_1, b_1)



$G(A, B)$

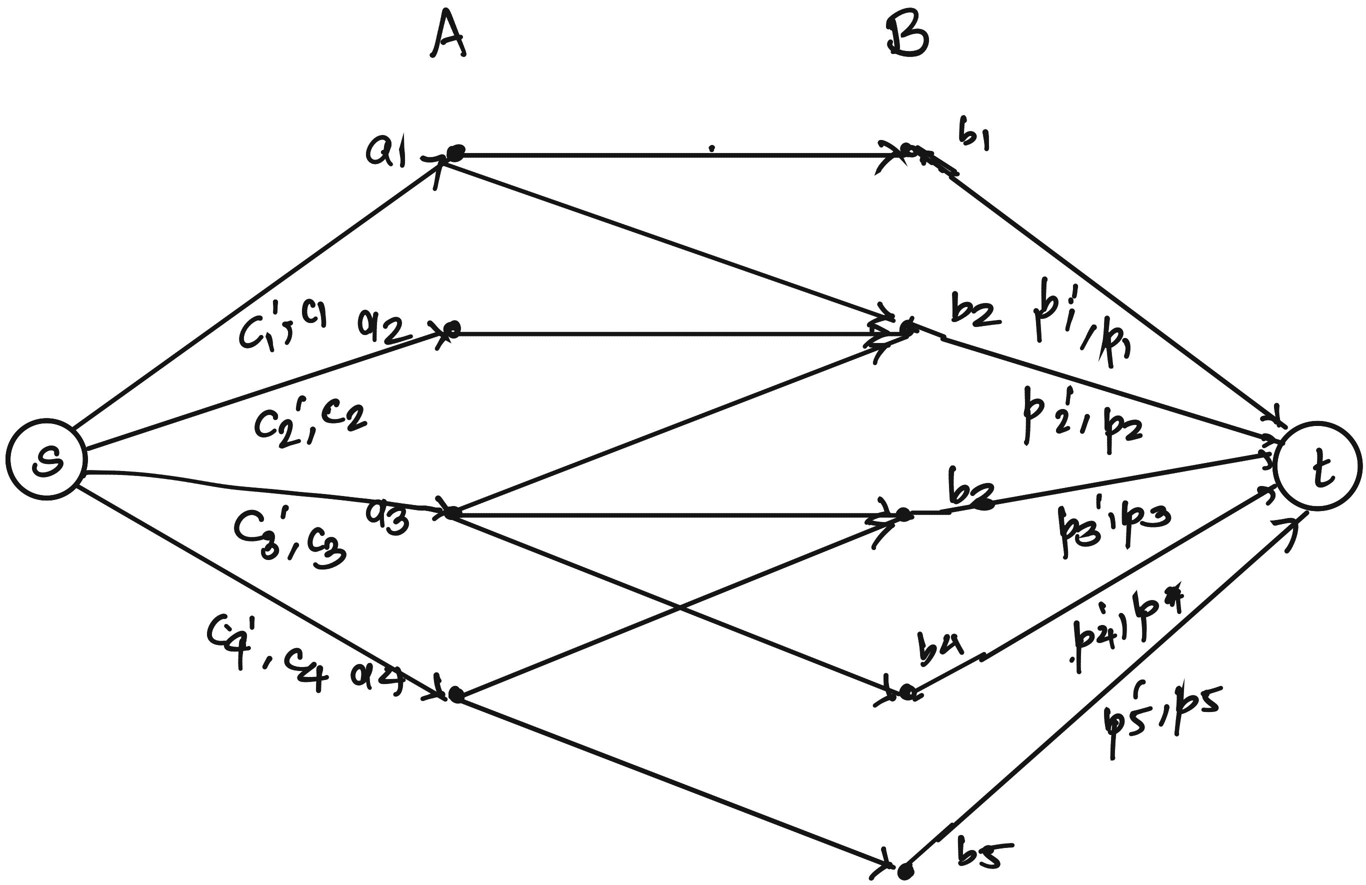
CUSTOMER PRODUCT.



CAN WE DESIGN A SURVEY FOR EACH CUSTOMER THAT SATISFY THESE CONDITIONS?

G(A, B)

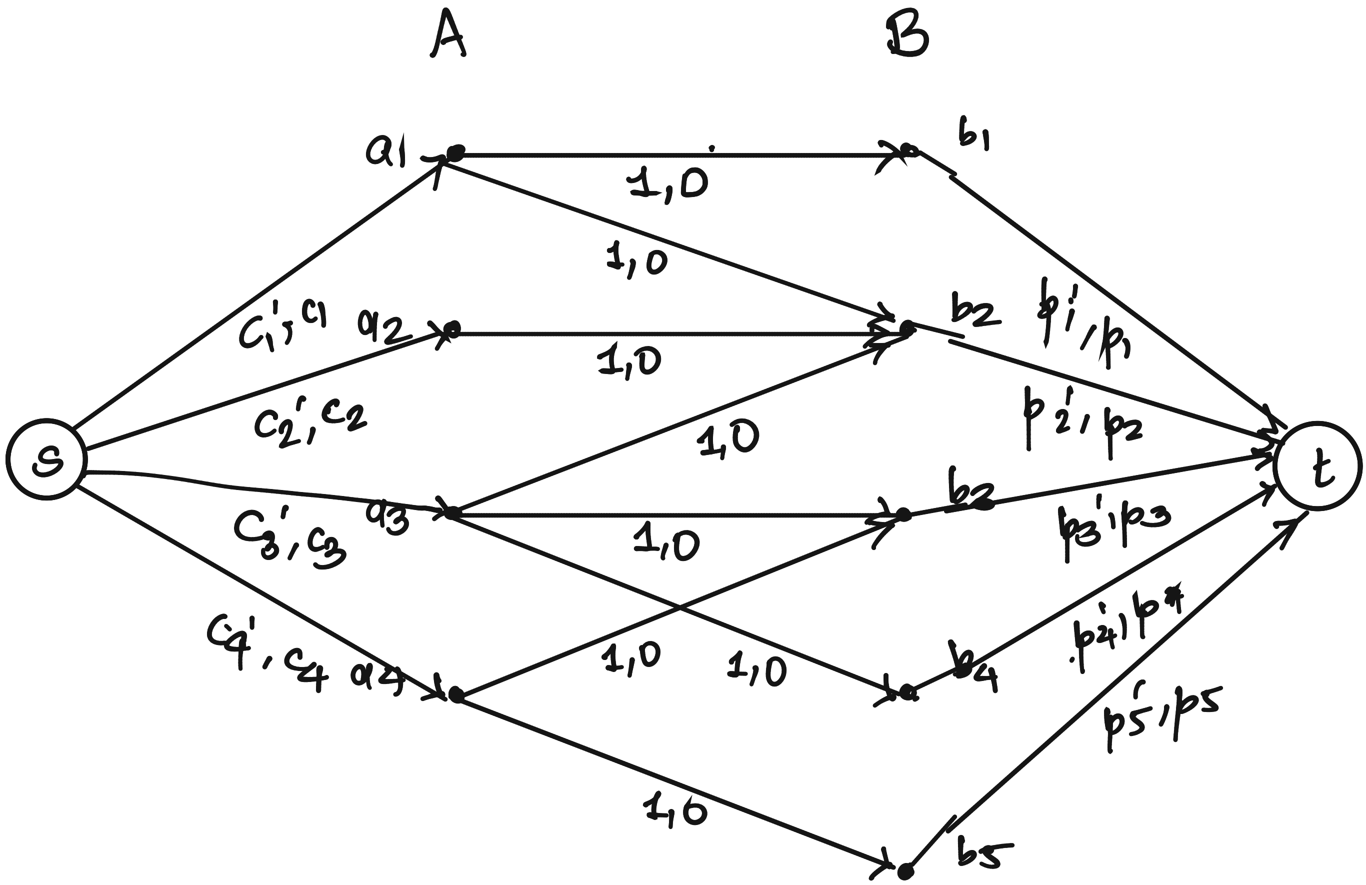
CUSTOMER PRODUCT.



CAN WE DESIGN A SURVEY FOR EACH CUSTOMER THAT SATISFY THESE CONDITIONS?

G(A, B)

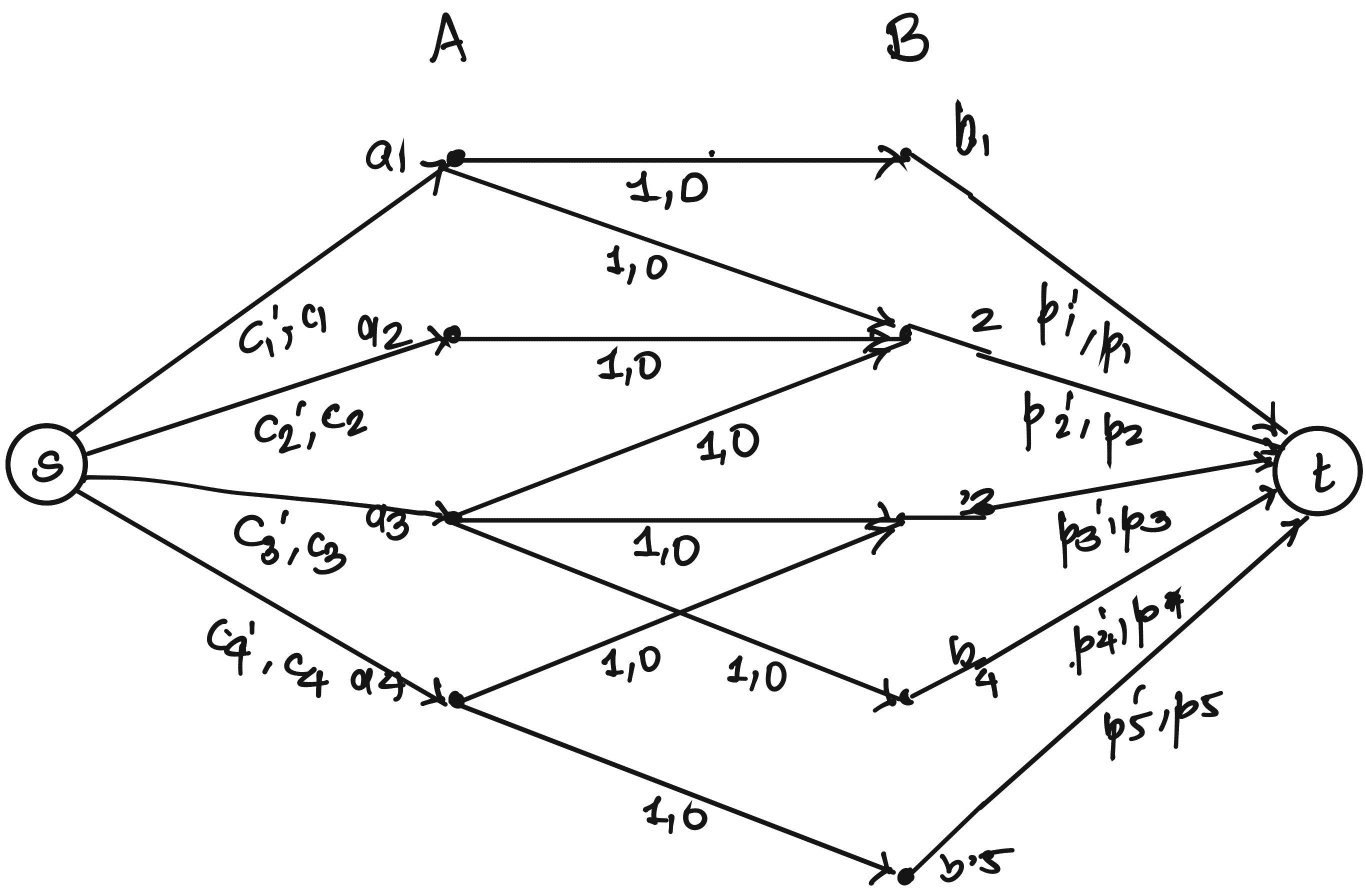
CUSTOMER PRODUCT.



CAN WE DESIGN A SURVEY FOR EACH CUSTOMER THAT SATISFY THESE CONDITIONS?

$G(A, B)$

CUSTOMER PRODUCT.

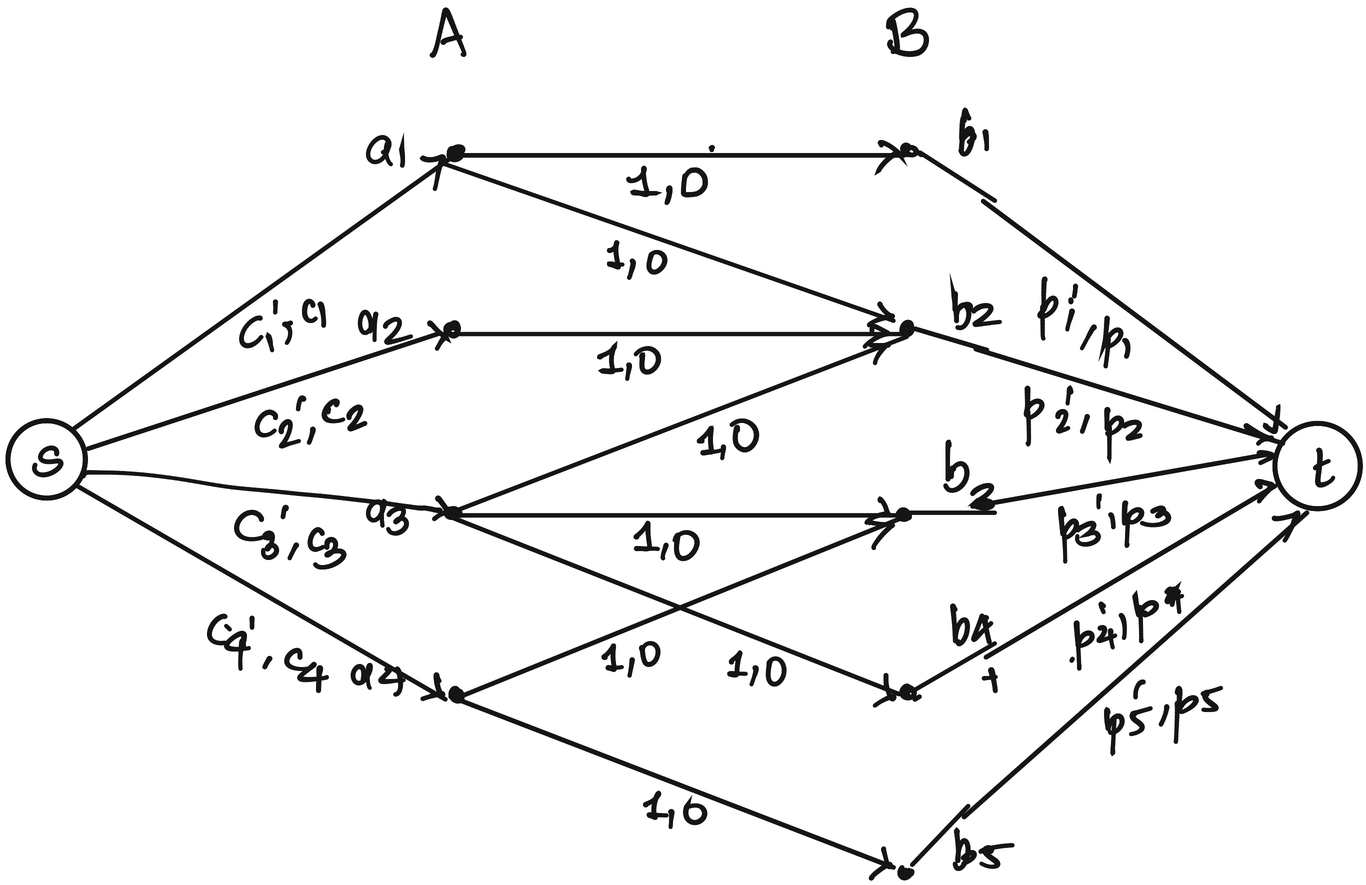


CAN WE DESIGN A SURVEY FOR EACH CUSTOMER THAT SATISFY THESE CONDITIONS?

Q: WHAT ABOUT d_v FOR EACH v?

$G(A, B)$

CUSTOMER PRODUCT.



CAN WE DESIGN A SURVEY FOR EACH CUSTOMER THAT SATISFY THESE CONDITIONS?

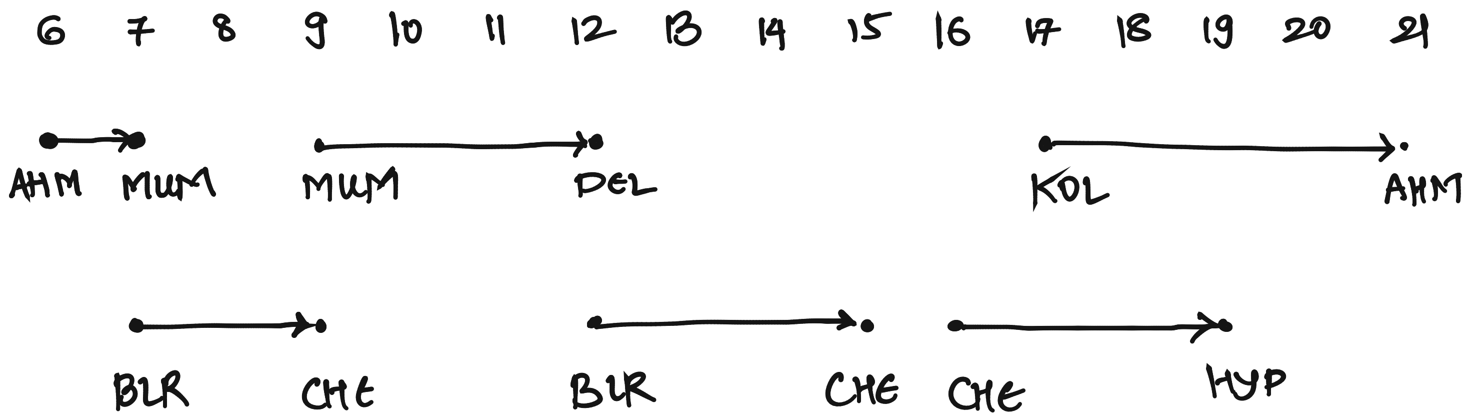
Q: WHAT ABOUT d_v FOR EACH v?

A: 0 FOR EACH v.

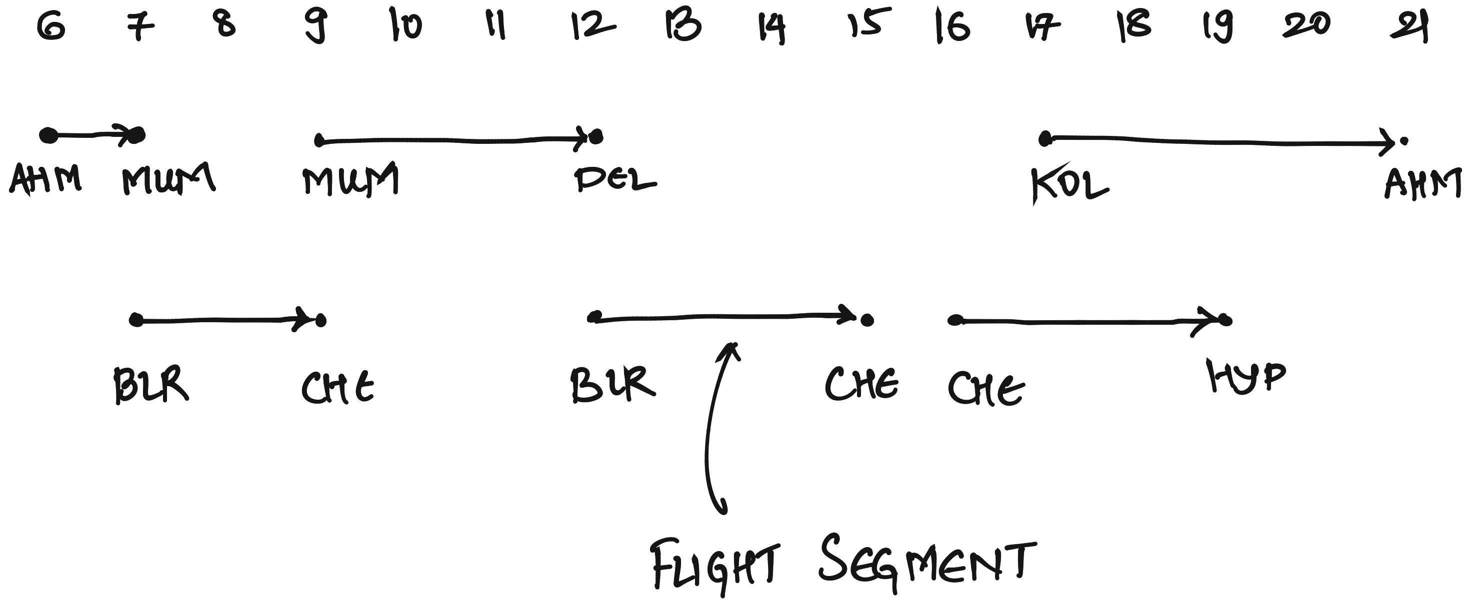
LEMMA: IF THERE IS A CIRCULATION IN G', THEN THERE IS A FEASIBLE WAY TO DESIGN SURVEY

⇒ **LEMMA**: IF THERE IS A FEASIBLE WAY TO DESIGN A SURVEY, THEN THERE IS A CIRCULATION IN G'.

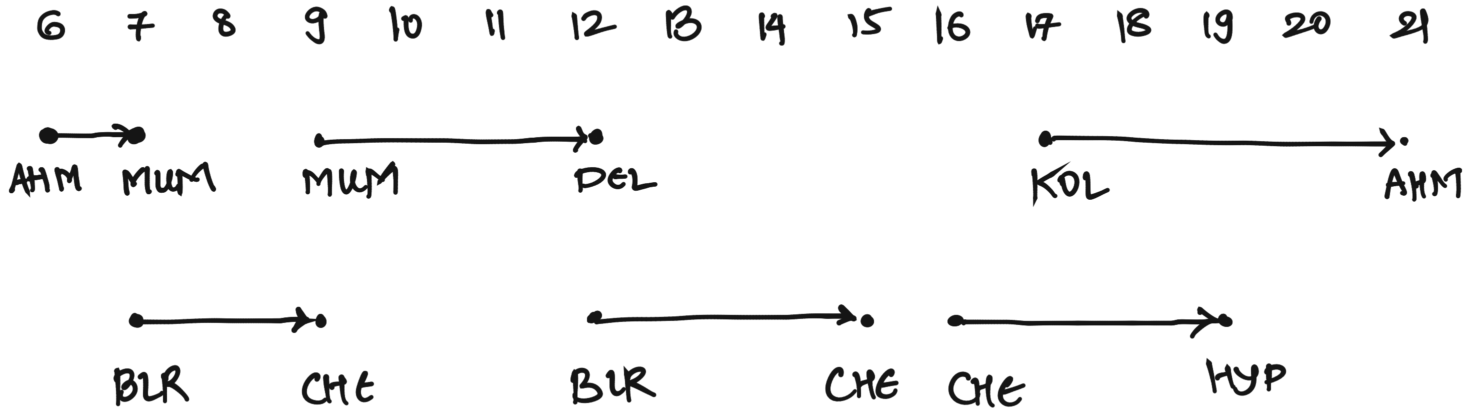
PROBLEM :


6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
AHM
MUM
MUM
DEL
KOL
AHM
BLR
CHE
BLR
CHE
CHE
HYD

PROBLEM :

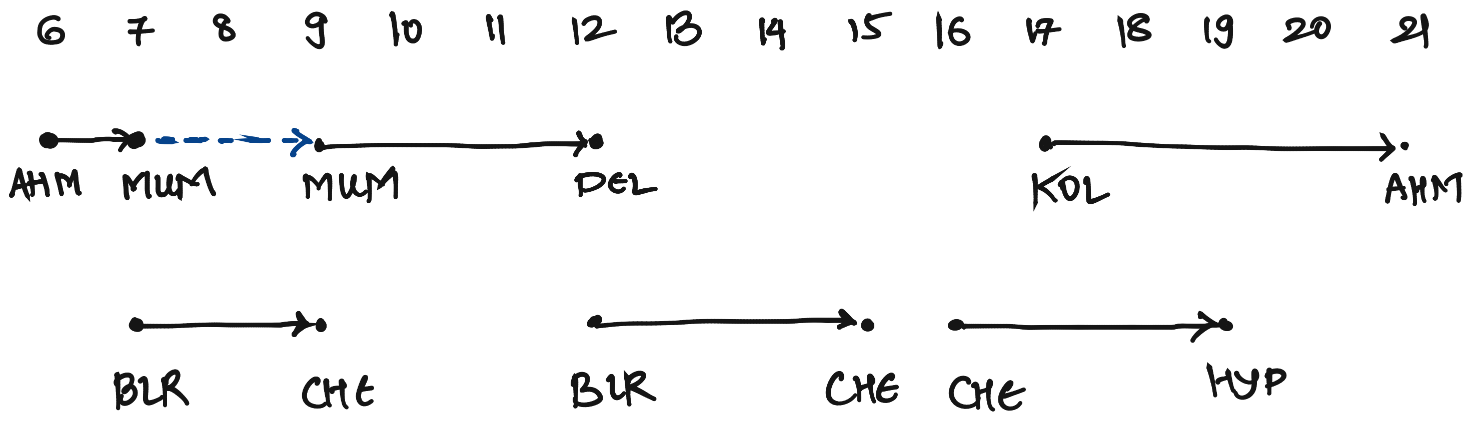

6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
AHM MUM
MUM DEL
KOL AHM
BLR CHE
BLR CHE
CHE HYD
FLIGHT SEGMENT

PROBLEM :



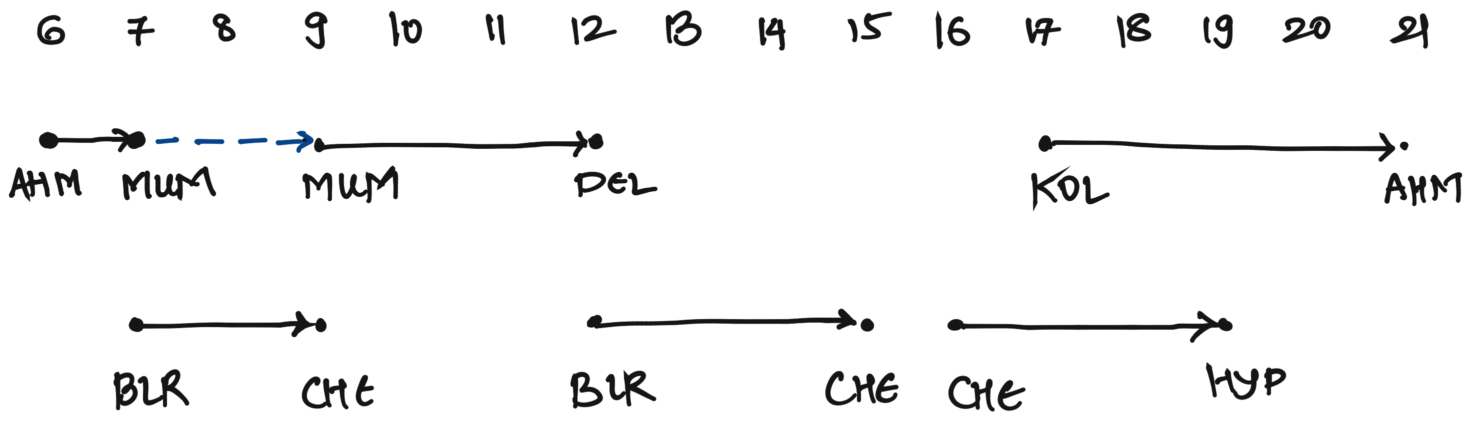
1) MAINTAINENCE TIME : 2 hour

PROBLEM :



1) MAINTAINENCE TIME : 2 hours

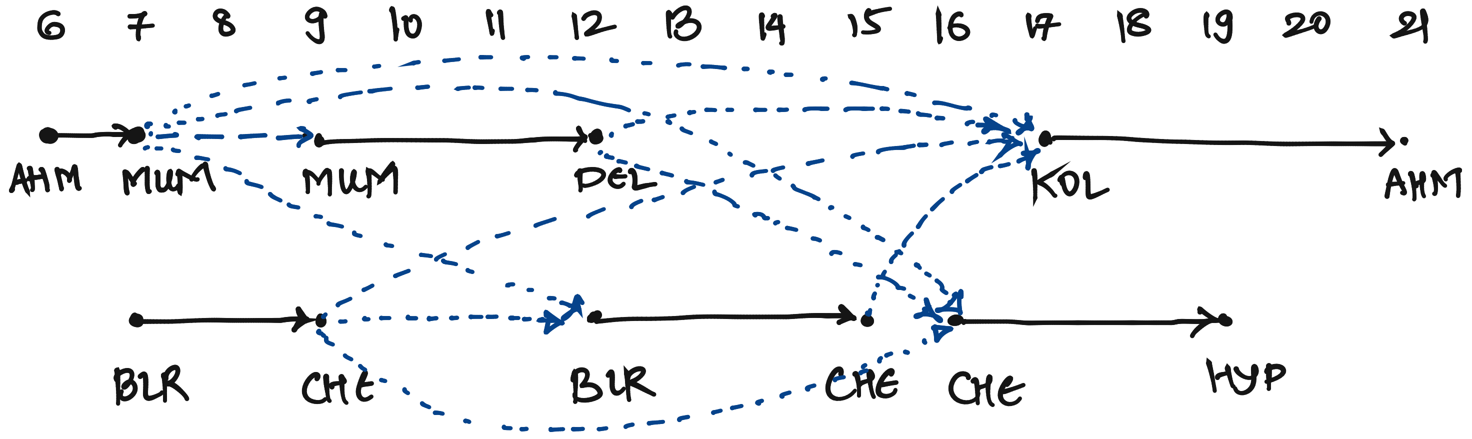
PROBLEM :



1) MAINTAINENCE TIME : 2 hour

2) FLIGHT SEGMENT j IS REACHABLE FROM FLIGHT SEGMENT i IF THE PLANE CAN GO FROM THE DESTINATION OF FLIGHT SEGMENT $i \rightarrow$ SOURCE OF FLIGHT SEGMENT j

PROBLEM :



1) MAINTAINENCE TIME : 2 hours

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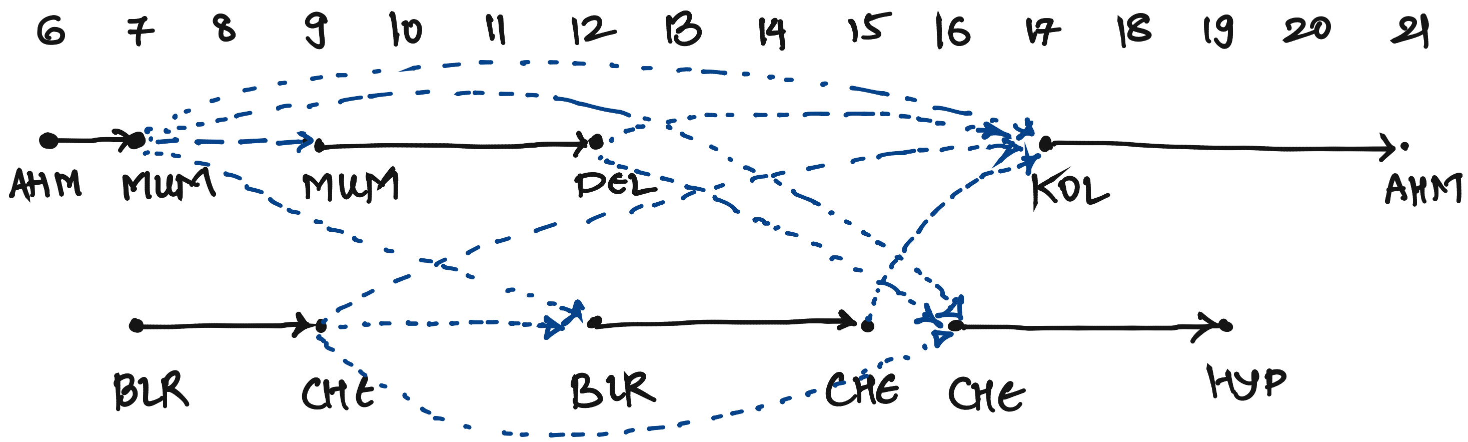
PROBLEM :

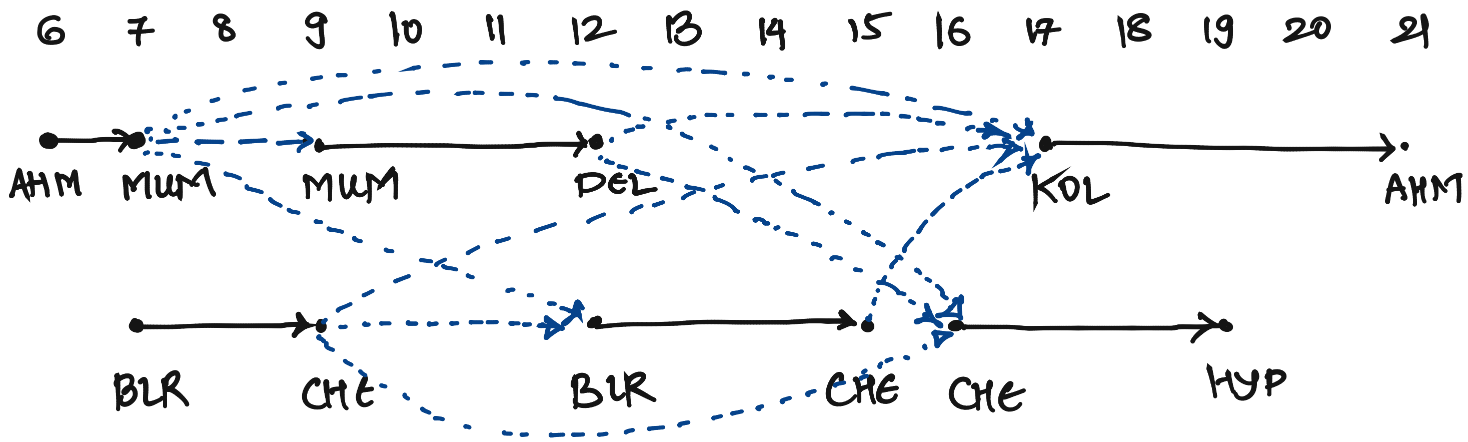


1) MAINTAINENCE TIME : 2 hour

2) FLIGHT SEGMENT j IS REACHABLE FROM FLIGHT SEGMENT i IF THE PLANE CAN GO FROM THE DESTINATION OF FLIGHT SEGMENT i → SOURCE OF FLIGHT SEGMENT j

Q: CAN WE COMPLETE ALL FLIGHT SEGMENTS USING TWO PLANES ?

PROBLEM :



1) MAINTAINENCE TIME : 2 hour

2) FLIGHT SEGMENT j IS REACHABLE FROM FLIGHT SEGMENT i IF THE PLANE CAN GO FROM THE DESTINATION OF FLIGHT SEGMENT i → SOURCE OF FLIGHT SEGMENT j

Q: CAN WE COMPLETE ALL FLIGHT SEGMENTS USING TWO PLANES ?

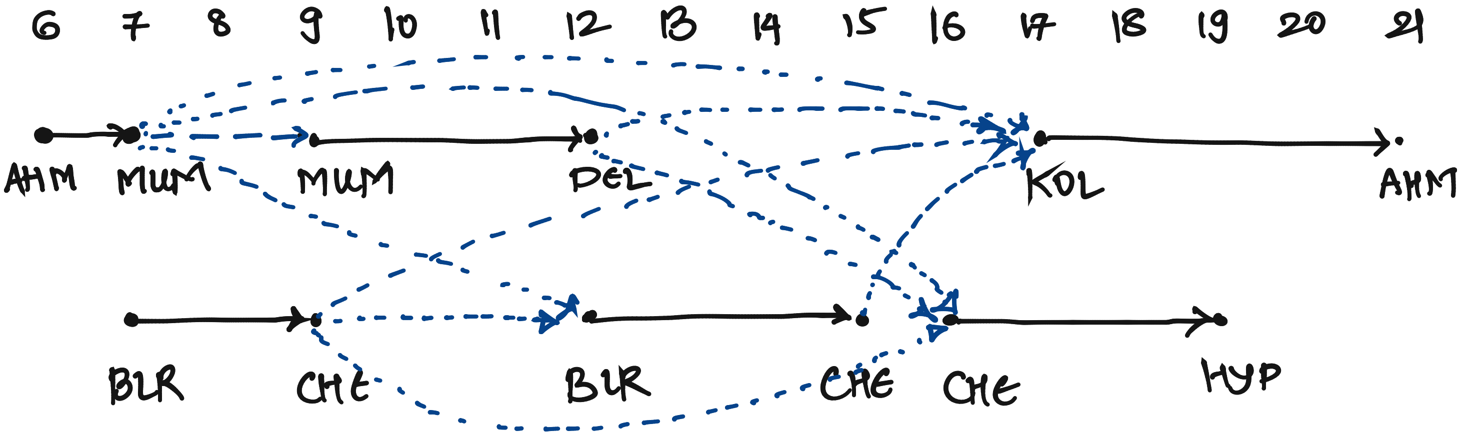
A: AHM - MUM MUM - DEL CHE - HYD
BLR - CHE BLR - CHE KOL - AHM

PROBLEM

FLIGHT SEGMENT : $1, 2, \ldots, n$

$i \longrightarrow j$ if A PLANE CAN REACH FROM DESTINATION OF SEGMENT i TO SOURCE OF SEGMENT j IN TIME

Q: CAN YOU SERVE ALL FLIGHT SEGMENTS USING K PLANES?


6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
AHM
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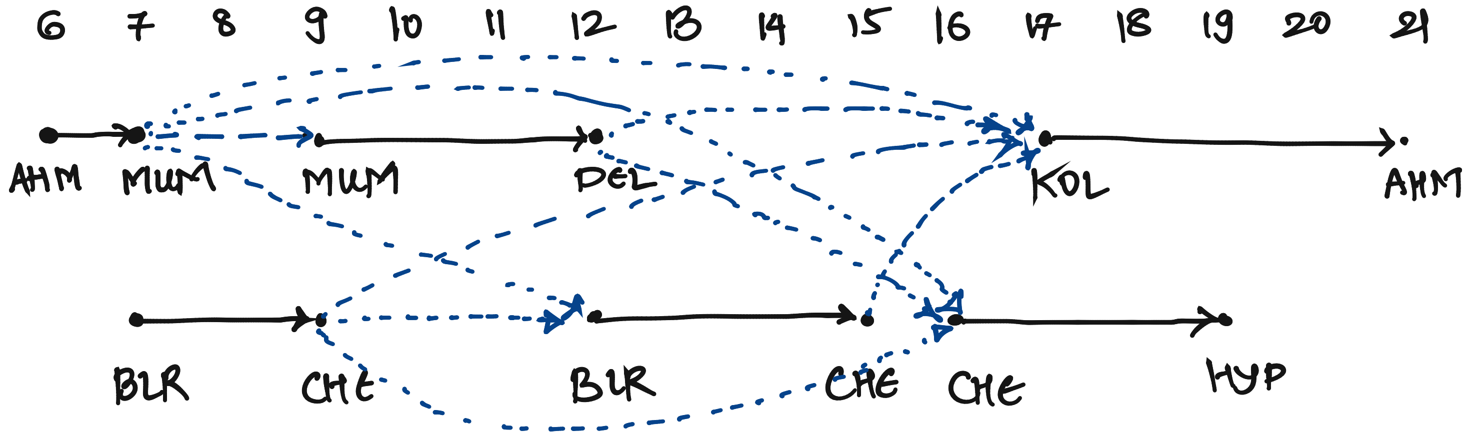
HYD

1 3 6

2 4 5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AHM
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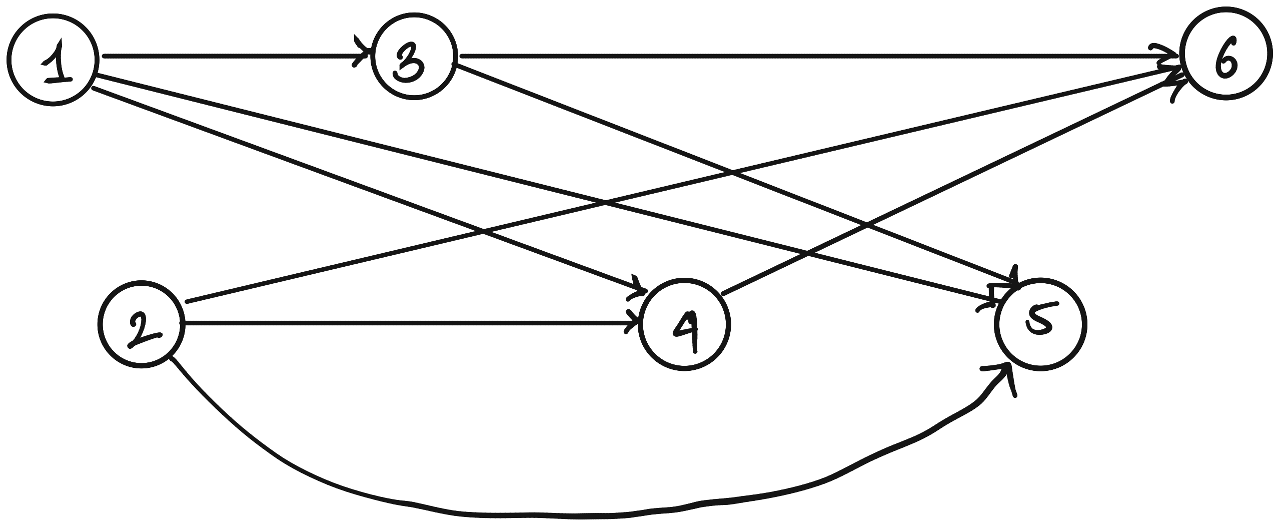
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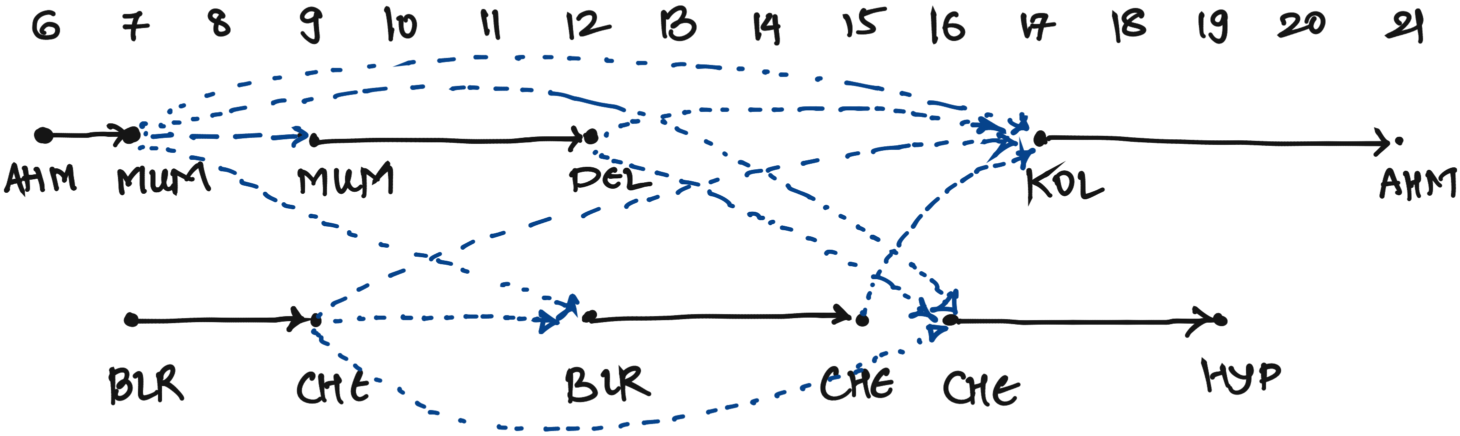
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AHM
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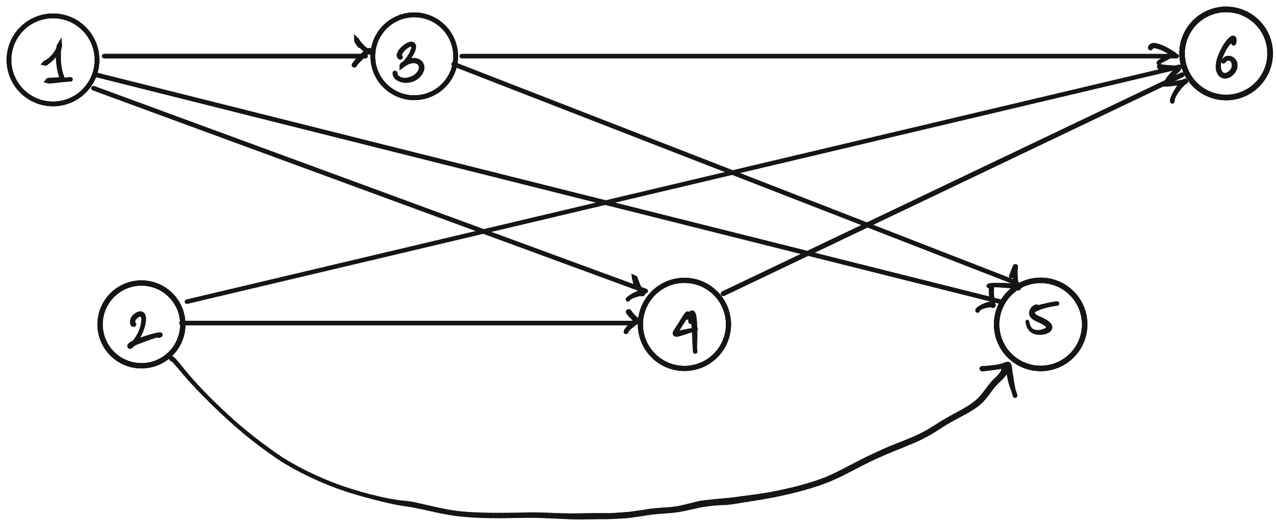
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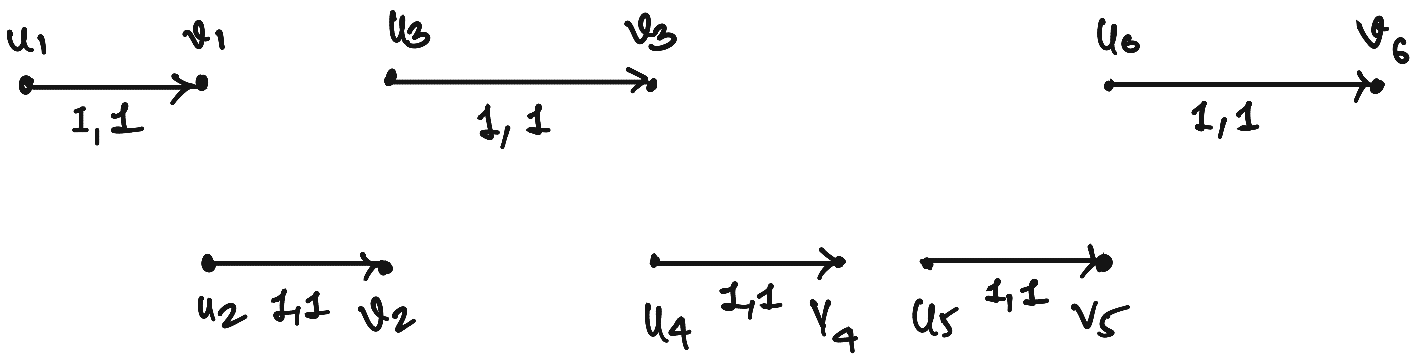
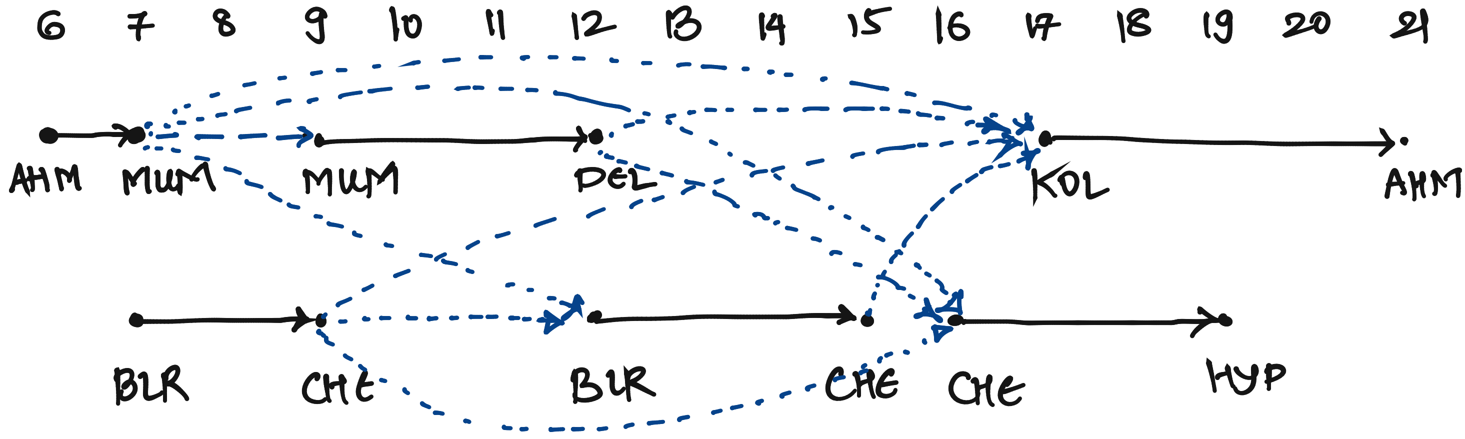

u1
v1
1, 1
u3
v3
1, 1
u6
v6
1, 1
u2
1,1
v2
u4
1,1
v4
u5
1,1
v5



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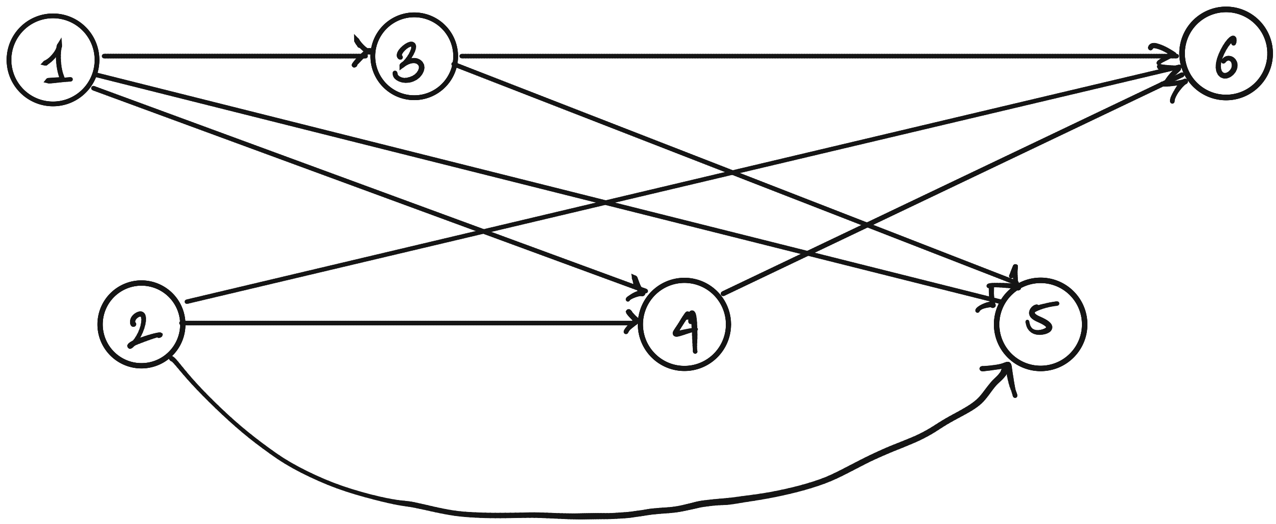

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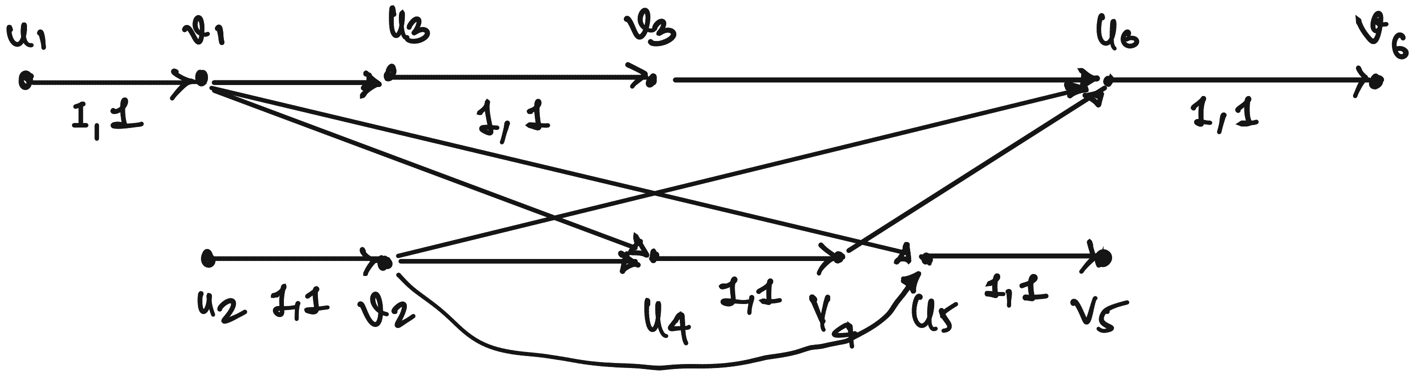

u1
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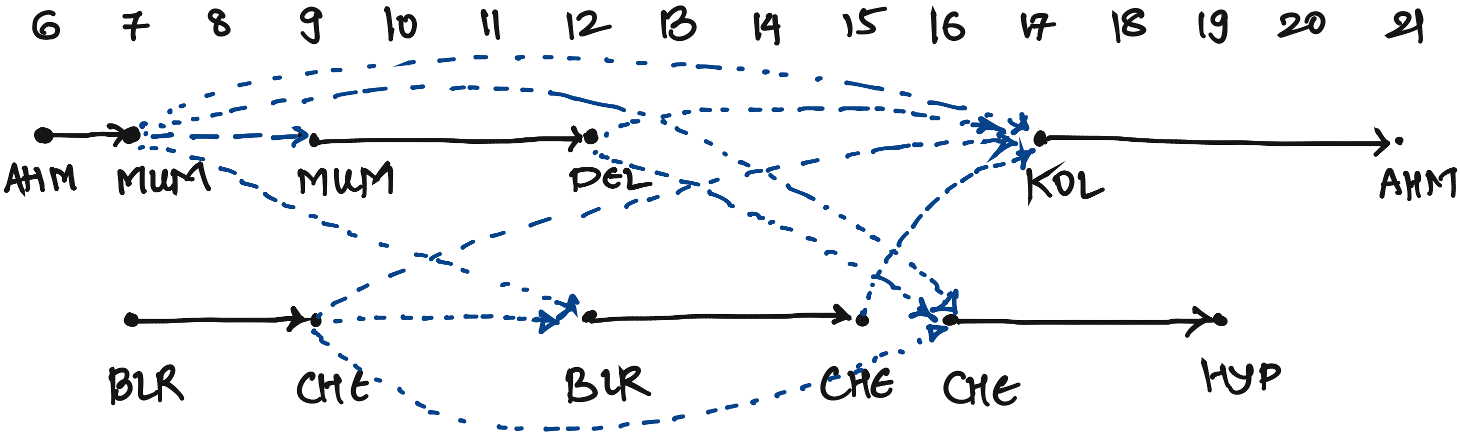
v5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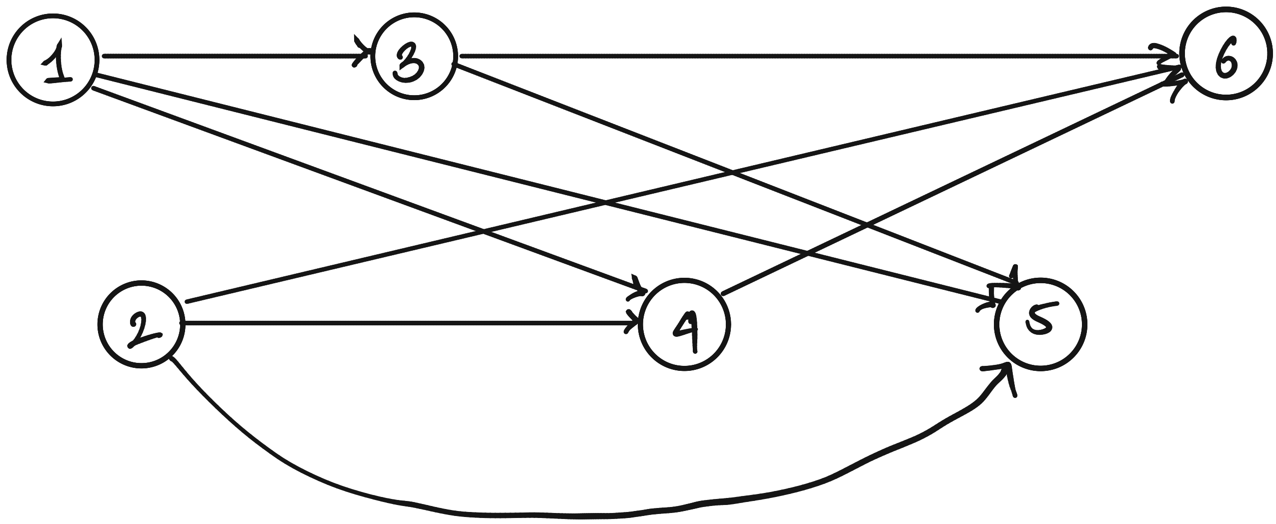
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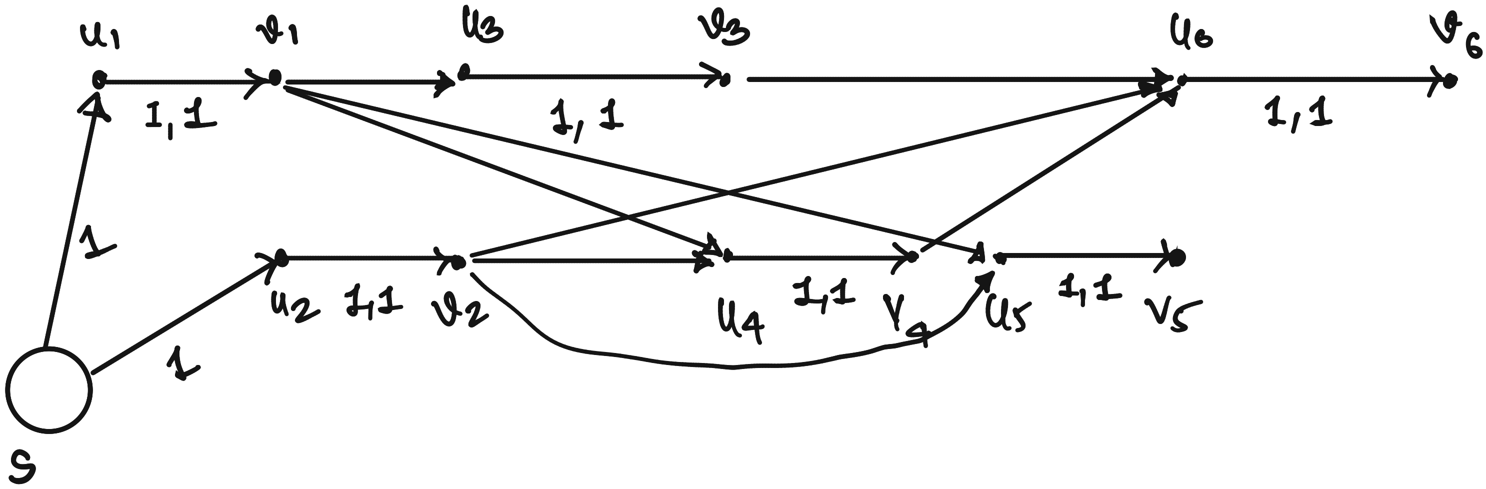
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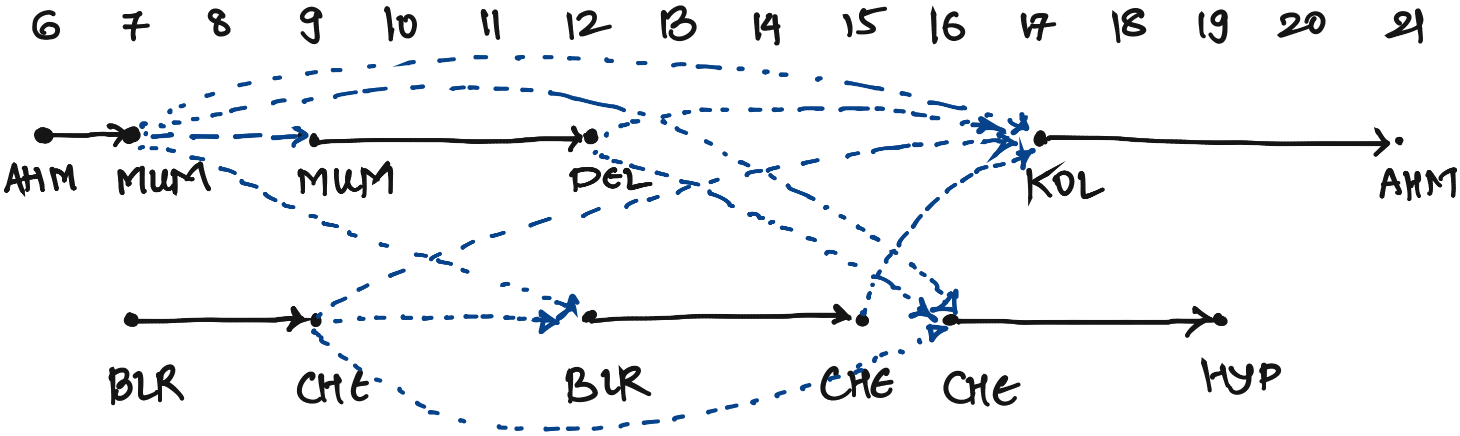
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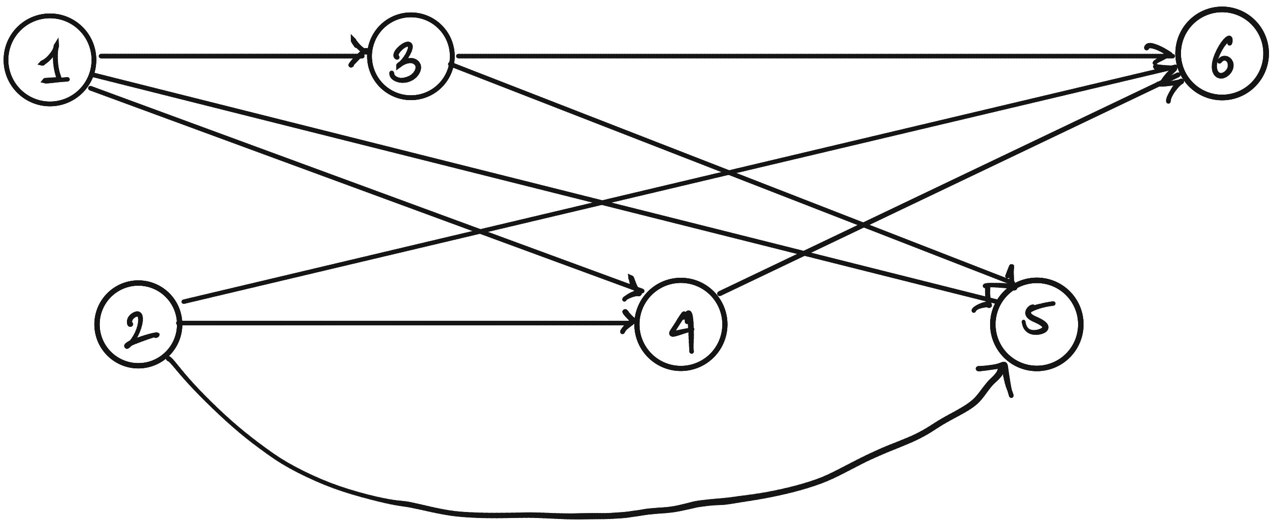
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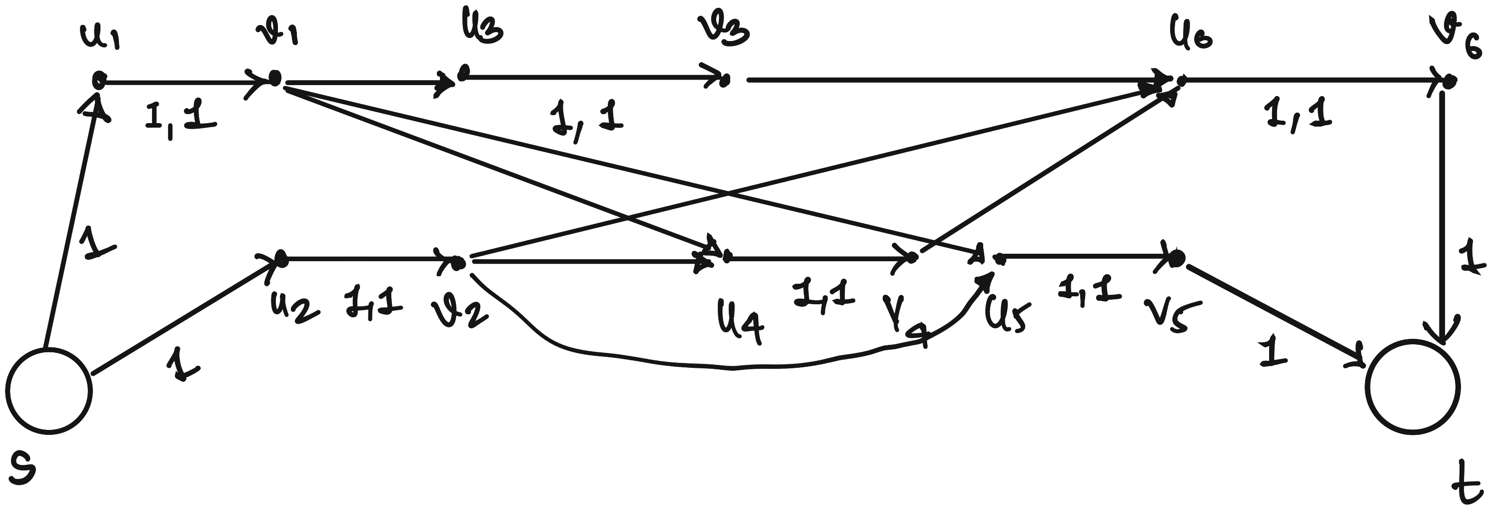
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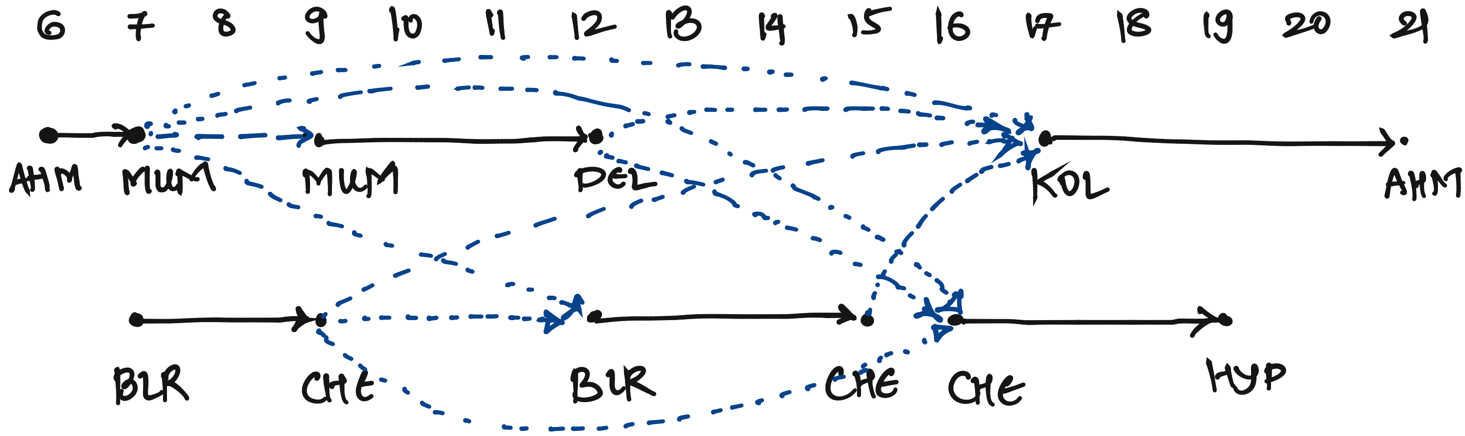
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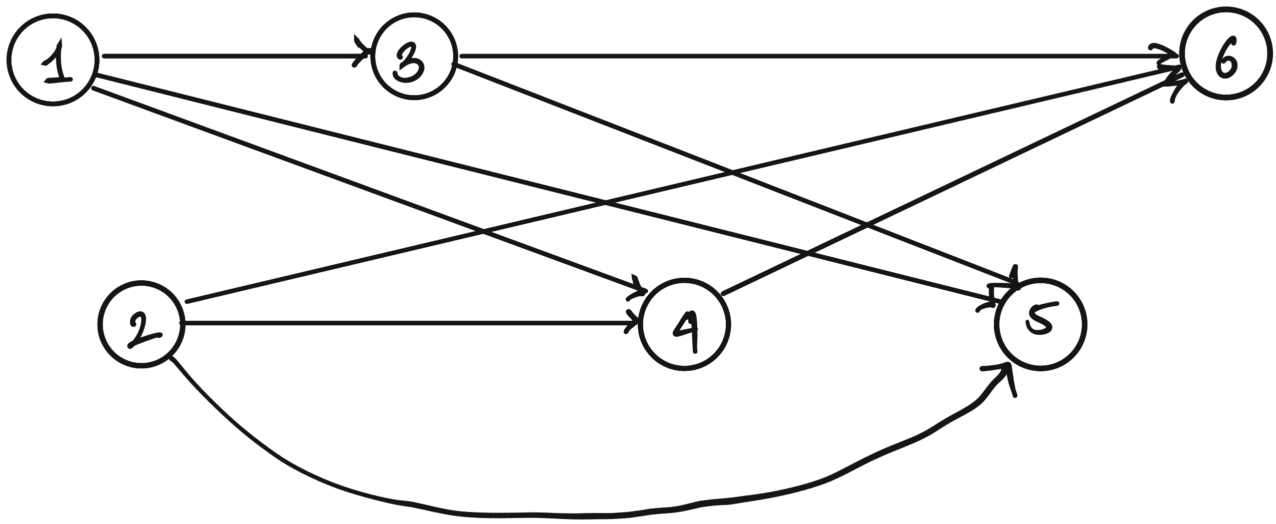
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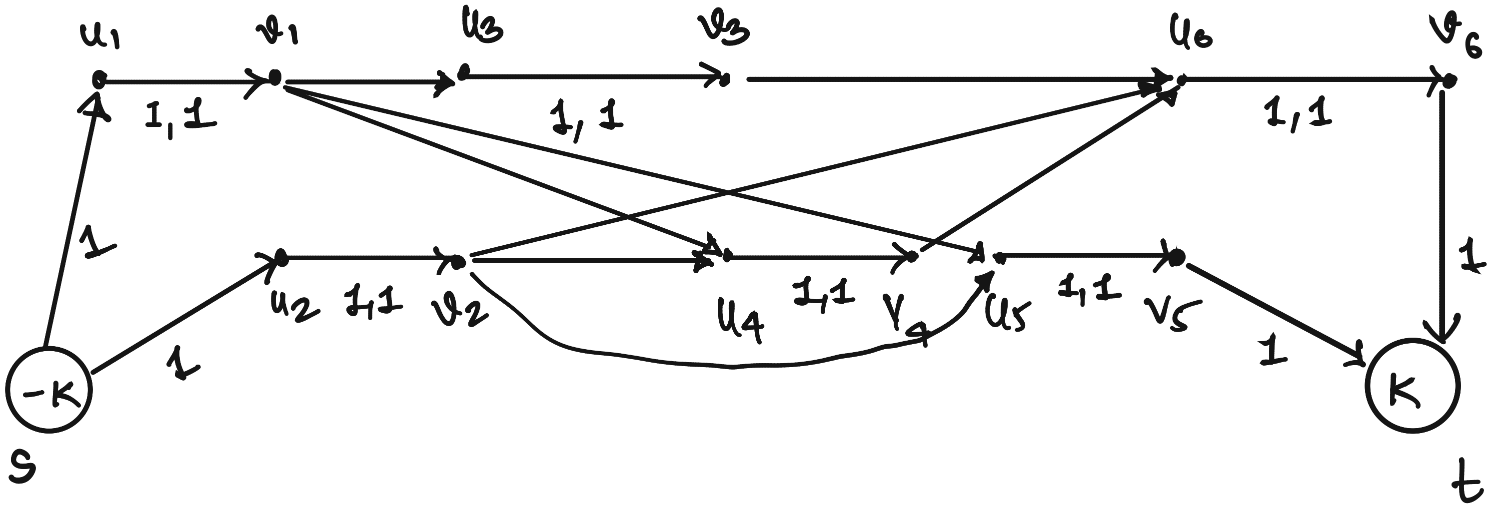
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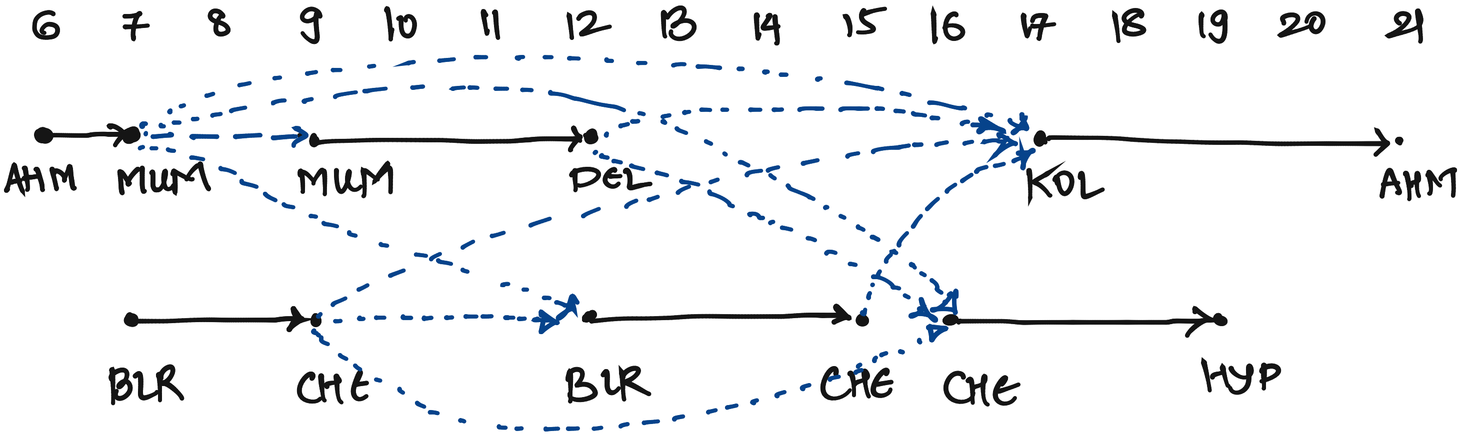
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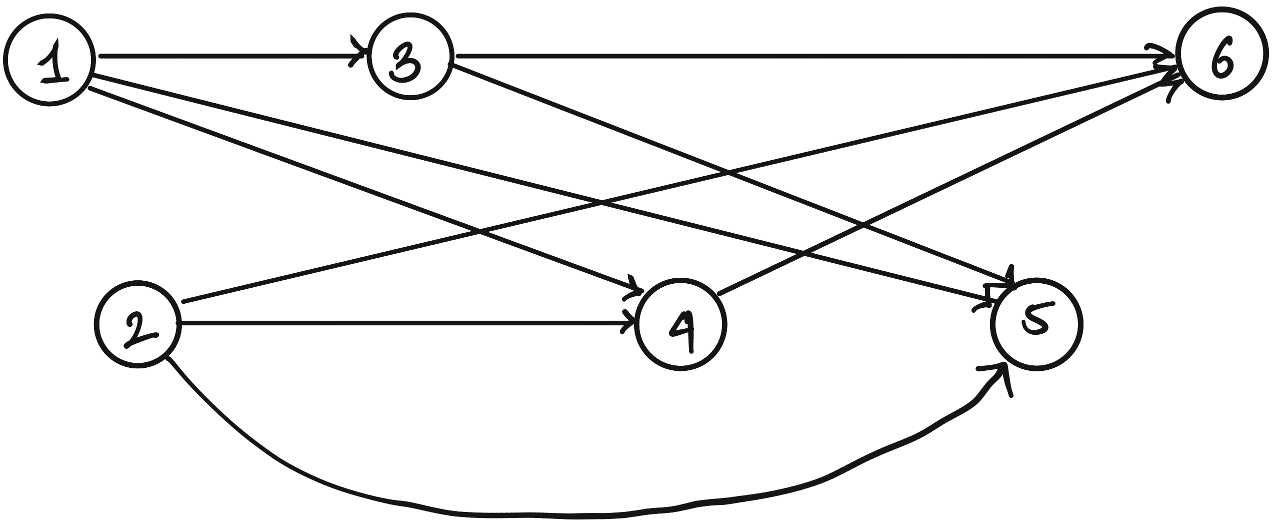
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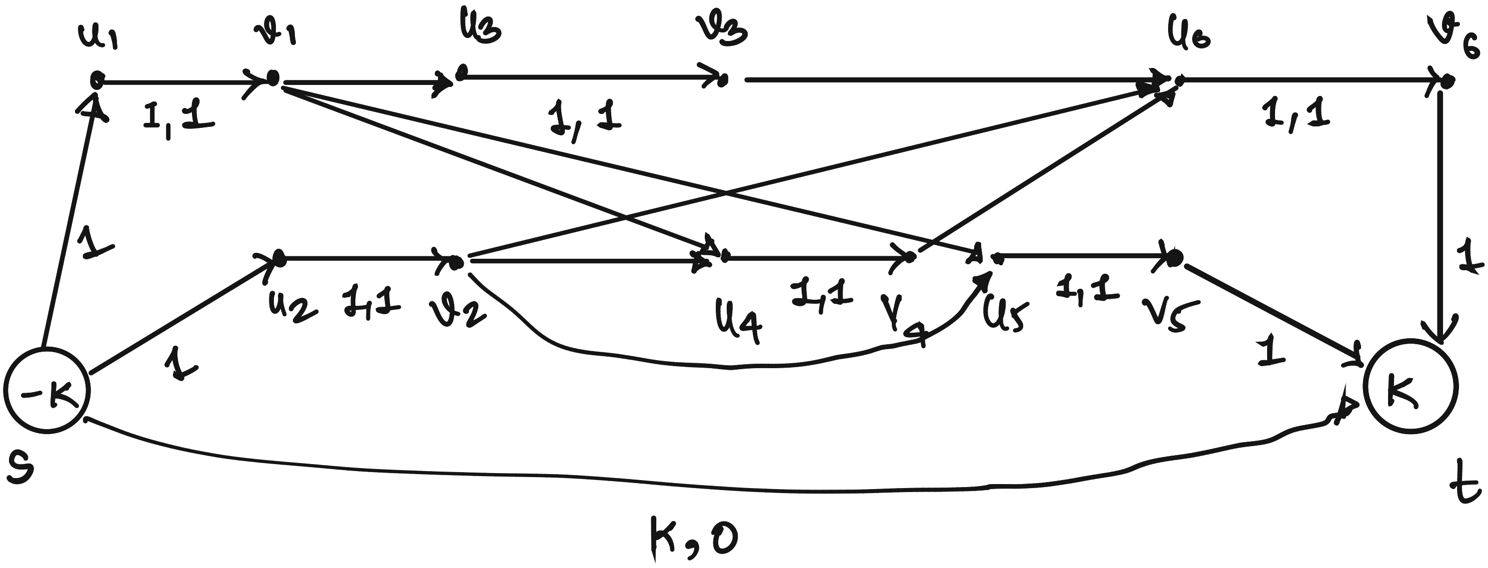
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u_4
1,1
v_4
u_5
1,1
v_5
1
1
-K
K
s
t
K, 0

1) FOR EACH SEGMENT i, $u_i \xrightarrow[1,1]{} v_i$

2) If FLIGHT SEGMENT j IS REACHABLE FROM FLIGHT SEGMENT i, $v_i \xrightarrow[1,0]{} u_j$

3) $s \xrightarrow[1,0]{} u_i$

4) $v_j \xrightarrow[1,0]{} t$

5) $s \xrightarrow[K,0]{} t$

6) s^{-K}

7) t^{K}

$\Rightarrow$ IF THERE IS A CIRCULATION IN G', THEN K PLANES CAN SATISFY ALL FLIGHT SEGMENTS

$\Leftarrow$ IF K PLANES CAN SATISFY ALL FLIGHT SEGMENTS, THEN THERE IS A CIRCULATION IN G'.

<u>PROBLEM</u>: GIVEN AN DIRECTED GRAPH G, WITH THREE VERTICES u, v & w, DESIGN AN ALGORITHM TO FIND IF THERE EXIST A PATH FROM u to v THAT PASSES THROUGH w.

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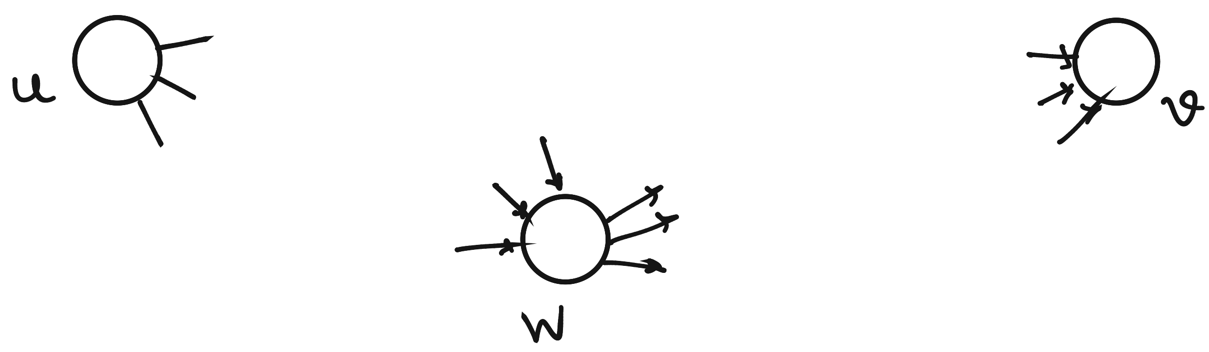

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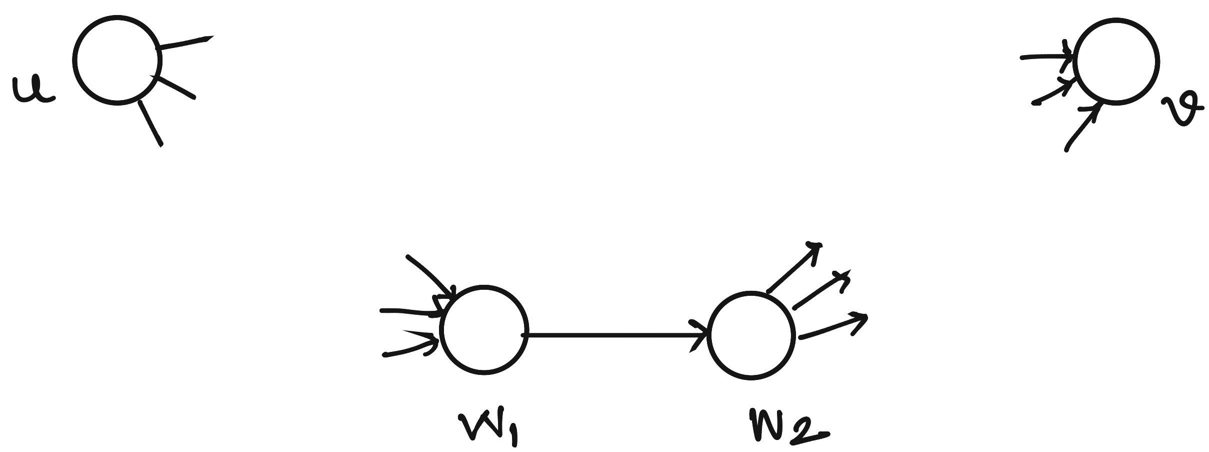
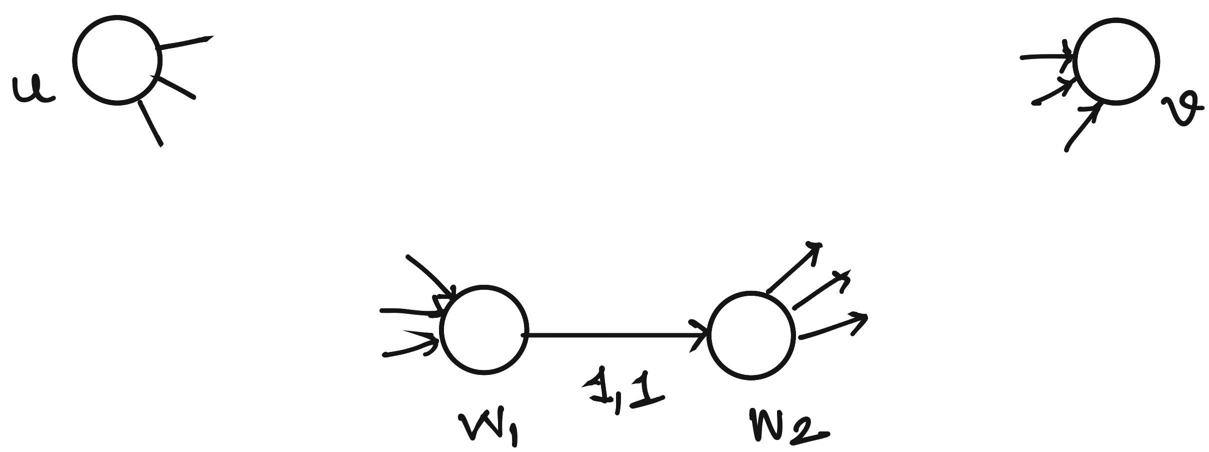


PROBLEM: GIVEN AN DIRECTED GRAPH G, WITH THREE VERTICES u, v & w, DESIGN AN ALGORITHM TO FIND IF THERE EXIST A PATH FROM u to v THAT PASSES THROUGH w.



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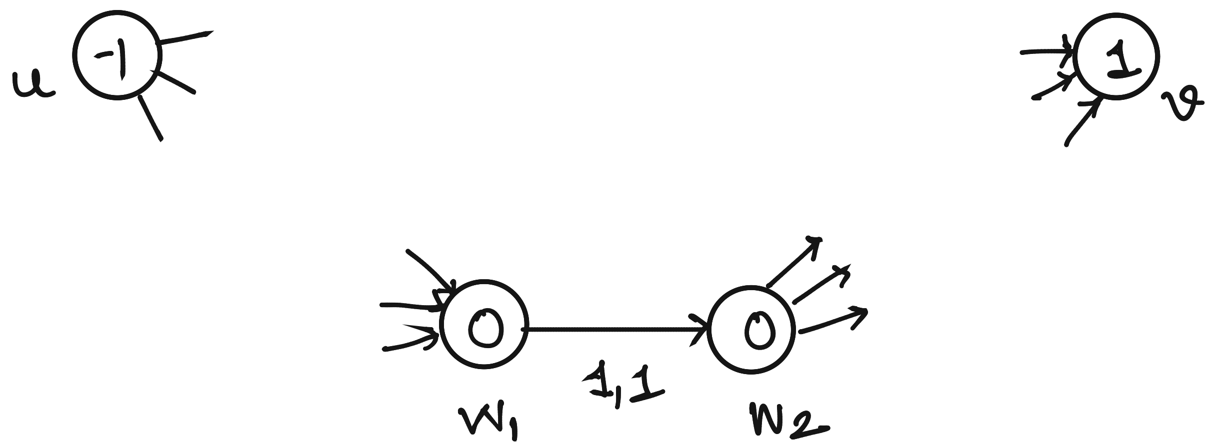
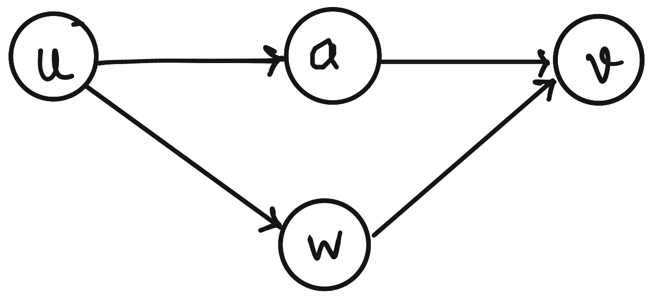
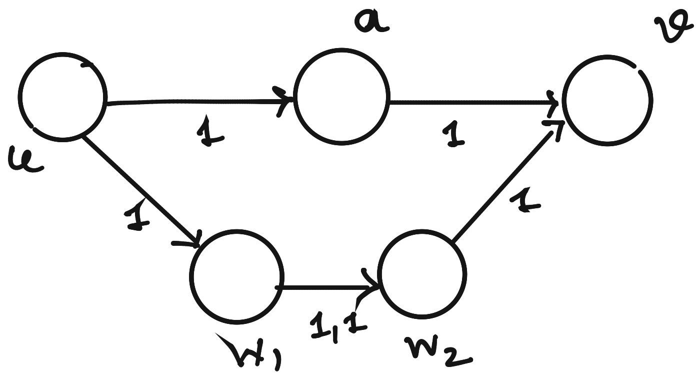
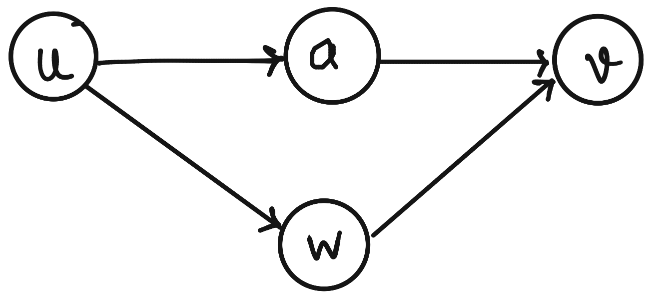


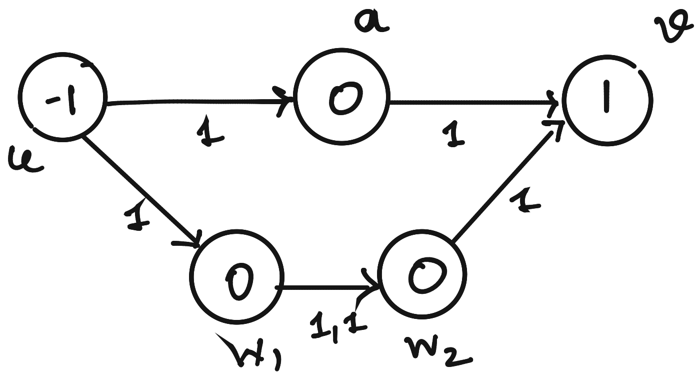


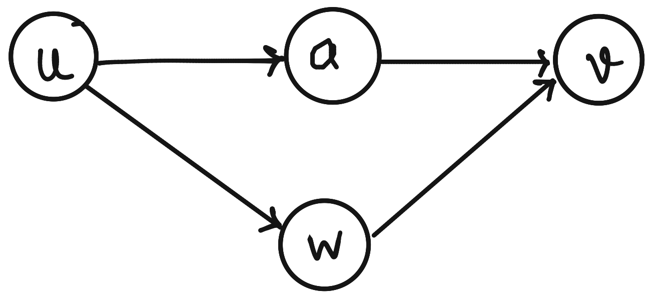
u
a
v
w

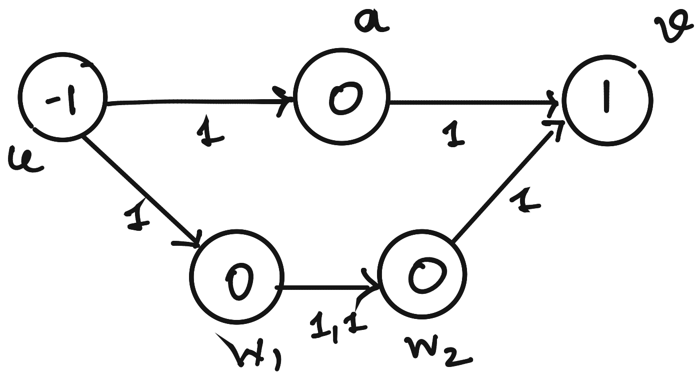


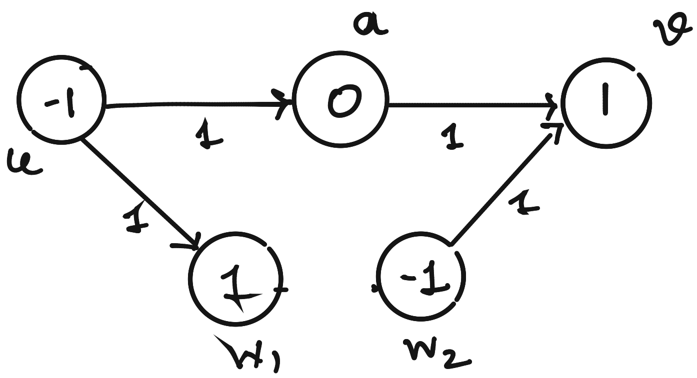
a
v
u
1
1
1
1
1,1
w1
w2

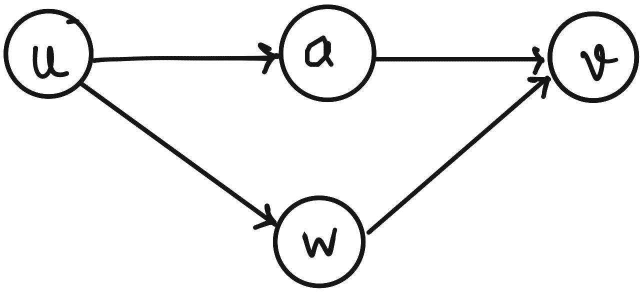

u
a
v
w


a
v
-1
0
1
u
1
1
1
1
0
0
1,1
w1
w2

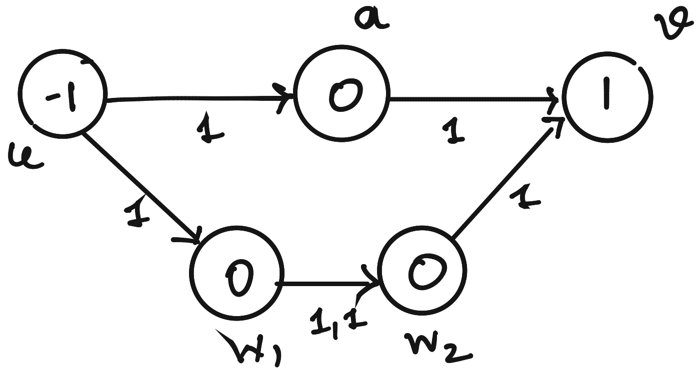

u
a
v
w


a
v
-1
0
1
u
1
1
1
1
0
0
1,1
w1
w2

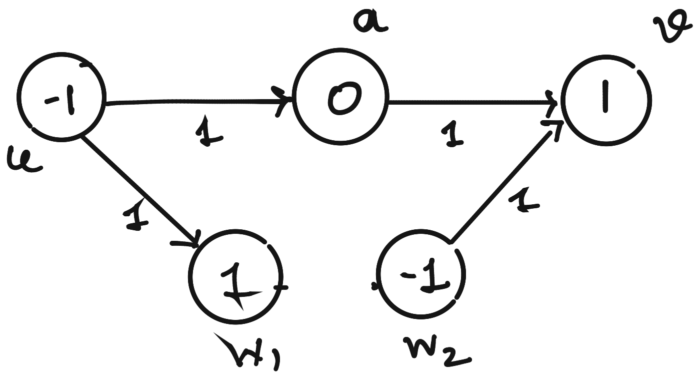

a
v
-1
0
1
u
1
1
1
1
1
-1
w1
w2



= OUR PROBLEM



= CIRCULATION WITH DEMAND & LOWER BOUND



= CIRCULATION WITH DEMAND

$\Updownarrow$

MAX FLOW

PROBLEM: ASSUME THAT YOU ARE GIVEN A FLOW IN G $\Rightarrow$ $f(e)$ FOR EACH e. DESIGN AN ALGORITHM TO FIND IF IT IS A MAX FLOW.

PROBLEM: ASSUME THAT YOU ARE GIVEN A FLOW IN G $\Rightarrow$ $f(e)$ FOR EACH e. DESIGN AN ALGORITM TO FIND IF IT IS A MAX FLOW.

ANS: REMEMBER THE FF ALGORITHM.

(1) GIVEN A VALID FLOW f, FIND A RESIDUAL GRAPH G_R.

(2) IF t IS REACHABLE FROM s IN G_R

PROBLEM: ASSUME THAT YOU ARE GIVEN A FLOW IN G $\Rightarrow$ $f(e)$ FOR EACH e. DESIGN AN ALGORITM TO FIND IF IT IS A MAX FLOW.

ANS: REMEMBER THE FF ALGORITHM.

(1) GIVEN A VALID FLOW f, FIND A RESIDUAL GRAPH G_R.

(2) IF t IS REACHABLE FROM s IN G_R
  f IS A MAX FLOW

  ELSE
  f IS NOT A MAX FLOW

PROBLEM: ASSUME THAT YOU ARE GIVEN A FLOW IN G $\Rightarrow$ $f(e)$ FOR EACH e. DESIGN AN ALGORITM TO FIND IF IT IS A MAX FLOW.

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f IS A MAX FLOW

ELSE

f IS NOT A MAX FLOW

RUNNING TIME: $O(m+n)$.

PROBLEM : → MOBILE COMPUTING CLIENTS ALWAYS NEED TO BE CONNECTED TO THE BASE STATIONS

→ ASSUME n MOBILE CLIENTS

→ K BASE STATIONS.

→ EACH MOBILE CLIENT & BASE STATION HAS A FIXED LOCATION

→ A MOBILE CLIENT CAN CONNECT TO A BASE STATION IF IT IS AT A DISTANCE $\leq r$ FROM THE BASE STATION

→ NO MORE THAN l CLIENTS CAN CONNECT TO A BASE STATION

DESIGN AN ALGORITHM TO DETERMINE WHETHER EACH CLIENT CAN CONNECT TO A BASE STATION UNDER ABOVE CONSTRAINTS.

FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

FOR EACH BASE STATION j, MAKE A NODE v_j

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

2) FOR EACH BASE STATION j, MAKE A NODE v_j

3) $u_i \longrightarrow v_j$, If DISTANCE BETWEEN u_i & $v_j \leq r$

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

2) FOR EACH BASE STATION j, MAKE A NODE v_j

3) $u_i \xrightarrow[1]{} v_j$, IF DISTANCE BETWEEN u_i & $v_j \le r$

4) $s \longrightarrow u_i$

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

2) FOR EACH BASE STATION j, MAKE A NODE v_j

3) $u_i \xrightarrow[1]{} v_j$, IF DISTANCE BETWEEN u_i & $v_j \leq r$

4) $s \xrightarrow[1]{} u_i$

5) $v_j \longrightarrow t$

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

2) FOR EACH BASE STATION j, MAKE A NODE v_j

3) $u_i \xrightarrow[1]{} v_j$, IF DISTANCE BETWEEN u_i & $v_j \leq r$

4) $s \xrightarrow[1]{} u_i$

5) $v_j \xrightarrow[l]{} t$

FIND A MAX FLOW IN G'.

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

2) FOR EACH BASE STATION j, MAKE A NODE v_j

3) $u_i \xrightarrow[1]{} v_j$, If DISTANCE BETWEEN u_i & $v_j \leq r$

4) $s \xrightarrow[1]{} u_i$

5) $v_j \xrightarrow[l]{} t$

FIND A MAX FLOW IN G'.

IF MAX FLOW IS OF VALUE ___,

1) FOR EACH MOBILE CLIENT i, MAKE A NODE u_i

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4) $s \xrightarrow[1]{} u_i$

5) $v_j \xrightarrow[l]{} t$

FIND A MAX FLOW IN G'.

IF MAX FLOW IS OF VALUE n, THEN THERE EXISTS A SOLUTION TO OUR PROBLEM.

$\Rightarrow$ <u>LEMMA</u> : IF THERE EXISTS A SOLUTION TO OUR PROBLEM, THEN THERE EXIST A MAX FLOW OF VALUE n IN G'.

$\Leftarrow$ IF THERE EXISTS A MAX FLOW OF VALUE n IN G', THEN THERE EXISTS A SOLUTION TO OUR PROBLEM.

<u>PROBLEM</u>: ASSUME THAT YOU ARE GIVEN A DIRECTED GRAPH WITH $C_e = 1$ ON EACH EDGE e.

YOUR GOAL IS TO DELETE A SET F OF EDGES SUCH THAT $|F| = k$. SUCH THAT THE MAX FLOW IN $G(V, E-F)$ IS MINIMIZED.

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A: IDEA: DECREASE THE SIZE OF MIN-CUT IN G.

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MAX FLOW IN G = WT. OF MIN CUT.

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A: IDEA: DECREASE THE SIZE OF MIN-CUT IN G.

MAX FLOW IN G = WT. OF MIN CUT.

USE FF ALGO TO FIND MIN-CUT IN G.

PROBLEM: ASSUME THAT YOU ARE GIVEN A DIRECTED GRAPH WITH $C_e = 1$ ON EACH EDGE e.

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RUNNING TIME:

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RUNNING TIME: $O(mn)$

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CORRECTNESS ??

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CORRECTNESS ??