## MA 201: Tutorial 2

1. Let $z = x + iy$. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though the Cauchy-Riemann equations are satisfied at that point.

2. Show that for the function $f(z) = u(x, y) + iv(x, y)$ given by

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0, \\ 0, & z = 0, \end{cases}$$

the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

3. Let $z = r(\cos\theta + i\sin\theta)$ and $f(z) = u(r, \theta) + iv(r, \theta)$. Derive the Cauchy-Riemann equations in the polar form given by

$$u_r = \frac{1}{r}v_\theta, v_r = -\frac{1}{r}u_\theta \quad (r > 0),$$

using the Cauchy-Riemann equations in the Cartesian coordinates.

4. Let $\mathfrak{D} = \{z \in \mathbb{C} : 0 < \mathrm{Re}(z) < 1\}$. Draw and explain the region where $\mathfrak{D}$ gets mapped to in the $w$-plane by $w = e^z$. Explain where the vertical and horizontal lines in $\mathfrak{D}$ get mapped.

5. Show that

$$\mathrm{Re}\tan z = \frac{\sin x\cos x}{\cos^2 x + \sinh^2 y},$$
$$\mathrm{Im}\tan z = \frac{\sinh y\cosh y}{\cos^2 x + \sinh^2 y}.$$

6. Prove that Im $\cos z$ and Re $\sin z$ are harmonic functions.

7. If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \dfrac{x}{x^2+y^2}$, determine $\phi$.