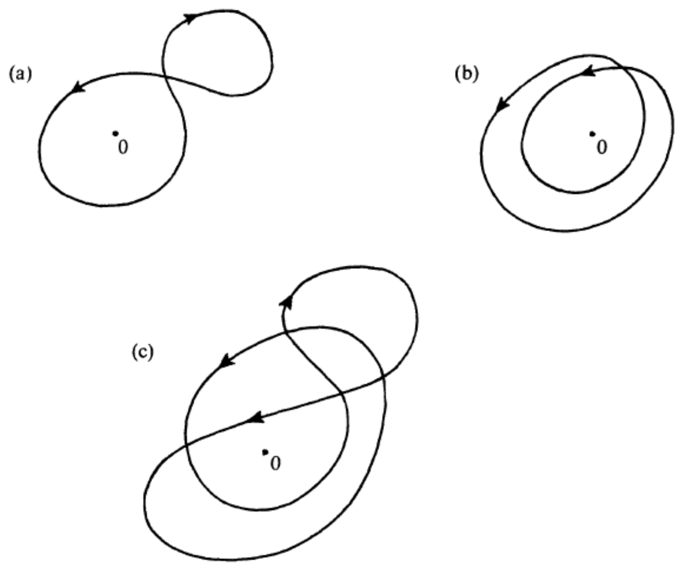
MA 502 - Assignment 2 (Complex Analysis)

1. Show that if $f : G \to \mathbb{C}$ is analytic and γ is a rectifiable curve in G, then $f \circ \gamma$ is also a rectifiable curve. (First assume G is a disk.)

2. Let $P : \mathbb{C} \to \mathbb{R}$ be defined by $P(z) = \text{Re}(z)$; show that P is an open map but is not a closed map. (Hint: Consider the set $F = \{z : \text{Im}(z) = \frac{1}{\text{Re}(z)} \text{ and } \text{Re}(z) \neq 0\}$).

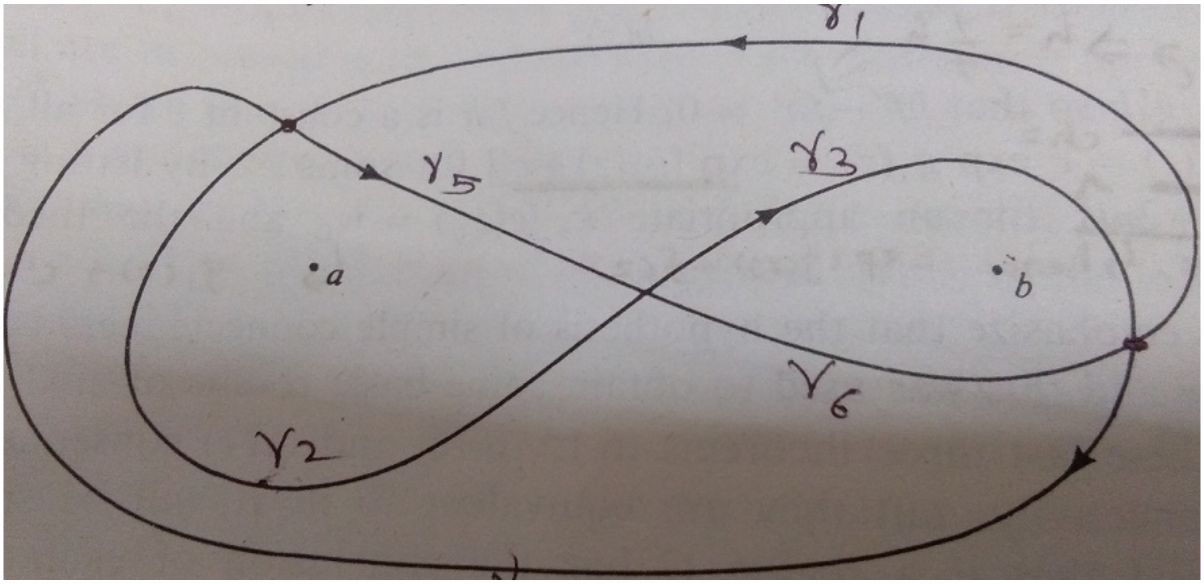
3. Let $\gamma(\theta) = \theta e^{i\theta}$ for $0 \leq \theta \leq 2\pi$ and $\gamma(\theta) = 4\pi - \theta$ for $2\pi \leq \theta \leq 4\pi$. Evaluate $\displaystyle\int_\gamma \frac{dz}{z^2 + \pi^2}$.

4. Evaluate $\displaystyle\int_\gamma \frac{e^z - e^{-z}}{z^4}\, dz$ where γ is one of the curves depicted below. (Justify your answer.)



5. Let X and Ω be metric spaces and suppose that $f : X \to \Omega$ is one-one and onto. Show that f is an open map iff f is a closed map. (A function f is a closed map if it takes closed sets onto closed sets.)

6. Let $G = \mathbb{C} - \{a, b\}, a \neq b$, and let γ be the curve in the figure below.



(a) Show that $n(\gamma; a) = n(\gamma; b) = 0$.

(b) Convince (not prove) yourself that γ is not homotopic to zero. Note that this example shows that it is possible to have a closed curve γ in a region such that $n(\gamma; z) = 0$ for all z not in G without γ being homotopic to zero.