MA 502 - Tutorial 10 (Complex Analysis)

1. Let G be open and suppose γ is a closed rectifiable curve in G. Let $r = d(\{\gamma\}, \partial G) > 0$.

(a) Show that $\{z : d(z, \partial G) < r/2\}$ is contained in the unbounded component of $\mathbb{C} - \{\gamma\}$.

(b) Use part (a) to show that if $f : G \to \mathbb{C}$ is analytic, then $f(z) = \alpha$ has at most a finite number of solutions z with $n(\gamma; z) \neq 0$.

2. Let f be analytic in $B(a; R)$ and suppose that $f(a) = 0$. Show that a is a zero of multiplicity m iff $f^{(m-1)}(a) = \cdots = f(a) = 0$ and $f^{(m)}(a) \neq 0$.

3. Suppose that $f : G \to \mathbb{C}$ is analytic and one-one; show that $f'(z) \neq 0$ for any z in G.

4. We have proved the following theorem in the class:

Let G be a region and let f be an analytic function on G with zeros $a_1 ..., a_m$ (repeated according to multiplicity). If γ is a closed rectifiable curve in G which does not pass through any point a_k and if $\gamma \sim 0$, then

$$\frac{1}{2\pi i} \int_\gamma \frac{f'(z)}{f(z)}\, dz = \sum_{k=1}^{m} n(\gamma; a_k).$$

Use this theorem to give another proof of the Fundamental Theorem of Algebra.