MA 502 - Tutorial 13 (Complex Analysis)

1. Prove that all four roots of $p(z) = z^4 + z - 7$ lie in $\text{ann}(0; 1, 2)$.

2. Prove that $\alpha z e^z = 1$ has exactly one root in $B(0; 1)$ for $|\alpha| > e$.

3. If A is a set dense in X, then A intersects every non-empty open set in X.

4. Suppose f is analytic on $\overline{B}(0; 1)$ and satisfies $|f(z)| < 1$ for $|z| = 1$. Find the number of solutions (counting multiplicities) of the equation $f(z) = z^n$ where n is an integer larger than or equal to 1.

5. Suppose that f is a meromorphic function on G. Define $f : G \to \mathbb{C}_\infty$ by setting $f(z) = \infty$ whenever z is a pole of f. Prove that f is continuous from G into $\mathbb{C}_\infty$.