MA 502 - Tutorial 8 (Complex Analysis)

1. Let G be a region and suppose that $f : G \to \mathbb{C}$ is analytic and $a \in G$ such that $|f(a)| \leq |f(z)|$ for all z in G. Show that either $f(a) = 0$ or f is constant.

2. If f is a non-constant analytic function on a bounded open set G and is continuous on $\overline{G}$, then show that either f has a zero in G or $|f|$ assumes its minimum value on ∂G.

3. Let G be a bounded region and suppose f is continuous on $\overline{G}$ and analytic on G. Show that if there is a constant $c \geq 0$ such that $|f(z)| = c$ for all z on the boundary of G, then either f is a constant function or f has a zero in G.

4. Suppose that both f and g are analytic on $\overline{B}(0; R)$ with $|f(z)| = |g(z)|$ for $|z| = R$. Show that if neither f nor g vanishes in $B(0; R)$ then there is a constant λ, $|\lambda| = 1$, such that $f = \lambda g$.

5. Let f be analytic in the disk $B(0; R)$ and for $0 \leq r < R$ define $A(r) = \max\{\text{Re}(f(z)) : |z| = r\}$. Show that unless f is constant, $A(r)$ is a strictly increasing function of r.