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MA 509 - REAL ANALYSIS - LECTURE 11

Some more definitions

Let X be a metric space.

(a) A neighborhood of p is

$$N_r(p) = \{ q : d(p,q) < r \text{ for some } r > 0 \}$$

radius

(b) Limit point p of a set $E \subset X$: is a point if every neighborhood of p contains a point $q \neq p$ such that $q \in E$.
(Also called an accumulation point)

(c) If $p \in E$ and p is not a limit point of E, then p is called an isolated point of E.

(d) E is closed if every limit point of E is a point of E.

Examples:

(e) A point p is an interior point of E if there is a neighborhood N of p s.t. $N \subset E$.

(f) E is open if every point of E is an interior point of E.

(g) Complement of E, denoted by E^c is
$E^c := \{x \in X : x \notin E\}$.

(h) E is perfect if E is closed and if every point of E is a limit point of E.

(i) E is bounded if $\exists$ $M \in \mathbb{R}$ & $q \in X$ $\ni$
$d(p,q) < M$ $\forall$ $p \in E$.

(j) E is dense in X if every point of X is a limit point of E, or a point of E (or both).

Theorem 2.8 Every neighborhood is an open set.
Proof: Let $E = N_r(p)$ be a nbhd of p and $q \in E \ni q \neq p$.
$\exists$ $h \in \mathbb{R}^+ \ni$ $d(p,q) = r - h$
Now if s is any point s.t. $d(q,s) < h$, then
$d(p,s) \leq d(p,q) + d(q,s) < r - h + h = r$.
$\Rightarrow s \in E$.
Thus, q is an interior point of E.