Thm. 2.9 If p is a limit point of a set E, then every nbhd of p contains infinitely many points of E.

Proof: Suppose $\exists$ a nbhd N of p containing only finitely many points of E, say, $q_1, q_2, \dots, q_n$, with $q_i \neq p$, $1 \le i \le n$.

Then put $r = \min_{1 \le i \le n} d(p, q_i)$.

Note that $r > 0$. (If it were, say, an infimum rather than a minimum, it could have taken the value 0.)

Then the nbhd $N_r(p)$ contains no point q of E $\ni q \neq p$, which contradicts our defn. of the limit point. $\blacksquare$

Cor. 2.10 A finite set has no limit points.

Examples Consider the following subsets of $\mathbb{R}^2$

a) $\{z \in \mathbb{C} : |z| < 1\}$
b) $\{z \in \mathbb{C} : |z| \le 1\}$
c) A finite set
d) $\mathbb{Z}$
e) $\{\frac{1}{n} : n \in \mathbb{N}\}$
f) $\mathbb{C}$ or $\mathbb{R}^2$
g) (a, b)

	Closed	open	perfect	bounded
a)	N	Y	N	Y
b)	Y	N	Y	Y
c)	Y	N	N	Y
d)	Y	N	N	N
e)	N	N	N	Y
f)	Y	Y	Y	N
g)	N	open in $\mathbb{R}$ but not in $\mathbb{R}^2$	N	Y

Thm. 2.11 Let $\{E_\alpha\}$ be a finite or an infinite collection of sets E_α. Then $\left(\bigcup_\alpha E_\alpha\right)^c = \bigcap_\alpha E_\alpha^c$.

Proof: Exercise.

Thm. 2.12 A set E is open iff its complement is closed.

Proof: Suppose E^c is closed. We want to show that E is open, i.e., every point of E is an interior point of E.

To that end, take a point $x \in E$. So $x \notin E^c$.

Now E^c being closed contain all its limit points. Hence, x cannot be a limit point of E^c. Therefore, $\exists$ nbhd U of x which does not intersect E^c at all, not even in x (since $x \notin E^c$).

Hence $U \cap E^c = \phi$ implies $U \subset E$ so that x is an interior point of E.

$\Rightarrow$ E is open.

The second half of the theorem will be done on Friday.