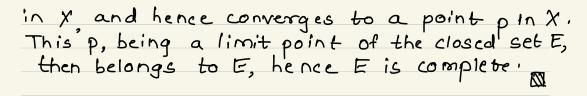
9/10/20

MA 509- REAL ANALYSIS-LECT. 24 Thm. B.G Let X be a metric space. Then a diam (E) = diam(E) b If Kn is a seq: of compact sets in X 3-Kn 2 Kn+1, neiN, and if lim diam Kn = 0, n+00 then MKn consists of a single point. Proof: (a) Done last time. (b) Let  $K = \bigcap_{n=1}^{\infty} K_n$ . Then K is non-empty, by Cor. 2:22. If k consists of more than one point, diam(K) >0. But Kn DK Unein, so diam(Kn), diam(K) Hnen. This contradicts the hypothesis that  $diam(Kn) \rightarrow 0$ . diam(Kn)→0, Thm. 3.7 a) In a metric space X, every Convergent seq: is Cauchy. (b) If X is a compact metric space, and if ipn } is a Cauchy sequence in X then ipn } converges to some point of X. C) In RK, every Cauchy sequence converges

Proof: ⓐ If 
$$p_n \rightarrow p$$
, given an  $2 > 0, 7$  Nervi  
>  $\gamma = n > N$ ,  $d(p_n, p) < 2/2$   
Then  $\forall = m, n > N$ ,  
 $d(p_n, p_m) \leq d(p_n, p) + d(p_m, p)$   
 $\leq \frac{e}{2} + \frac{e}{2}$   
 $\Rightarrow \qquad 2p_n r is Cauchy sequence in the compact-
metric space  $\chi$ . For NGN, let  
EN =  $\{P_N, P_{N+1}, P_{N+2}, \dots, \}$ .  
Then by defn. of diameter of a set, and part ⓐ  
of the previous theorem, we have  
 $\lim_{n \to \infty} diam(E_N) = 0$ . (i)  
N→∞  
Each  $E_N$  is closed subset of the compact-  
Gace  $\chi$ , hence compact. (ii)  
Also  $E_N \supset E_{N+1}$ , so that  $E_N \supset E_{N+1}$  (iii)  
From (i), (ii)  $\mathcal{A}$  (iii),  $\bigcap_{n \in I} E_n$  is a singleton set; let  
 $P \in E_N \forall N \in N$ .  
Let  $2 > 0$  be given. From (i),  $7 = N_0 \in N \Rightarrow$   
diam( $E_N$ ) <  $\xi$  for  $N > N_0$ .  
Since  $p \in E_N$ ,  $d(p,q) < \xi$  for every  $q \in E_N$ ,$ 

and hence for every QEEN. Thus, d(p,pn)< & for n>, No. => {pn}->p.  $\boxtimes$ ELet EPn3 be a Cauchy sequence in R<sup>k</sup>. Suppose EN= 2 PN PN+1, PN+2 for NEN. Since { pn } is Cauchy, diam EN -> 0 cg N-> 0. Hence J N > diam (EN) < 1. The range of {Pn} is ENU {P1, P2, ..., PN-1}. ⇒ {Pn} is bounded. d(P1, P2, ..., PN-1}. So can take, say, 2 man {1, d(P1, P2).... d (PN-1, PN-2), d(P1, PN)), upper Since every bounded set of IR has compare closure in IRK, © now follows from (b). COMPLETE METRIC SPACE A metnic space is said to be <u>complete</u> if every Cauchy sequence in it converges. e.g. All compact metric spaces, R<sup>k</sup> are Complete. Remark Every closed subset of a complete metric space is complete. Proof: Let & pnz be a Cauchy sequence in the closed subset E of X. Thus it's a Cauchy seq.



· example of a metric space which is nobcomplete: (Q,d) with d(xy) = 1x-y].