MA 509: Tutorial 12 (2020)

1. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

2. Consider

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2x}.$$

For what values of x does the series converge absolutely? On what intervals does it converge uniformly? On what intervals does it fail to converge uniformly? Is f continuous wherever the series converges? Is f bounded?