MA 509: Tutorial 1 (2020)

1. Prove that there is no rational number whose square is 12.

2. Find the greatest lower bounds of the following sets:

   (a) $(7, 8)$.
   (b) $\{\pi + 1, \pi + 2, \pi + 3, \cdots\}$.
   (c) $\{\pi + 1, \pi + \frac{1}{2}, \pi + \frac{1}{3}, \cdots\}$.

3. Show that the axioms of multiplication imply the following statements:

(a) If $x \neq 0$ and $xy = xz$, then $y = z$.

(b) If $x \neq 0$ and $xy = x$, then $y = 1$.

(c) If $x \neq 0$ and $xy = 1$, then $y = 1/x$.

(d) If $x \neq 0$, then $1/(1/x) = x$.

4. Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that

$$\inf(A) = -\sup(-A).$$

5. If A is a nonempty bounded subset of an ordered set S, and $\inf(A) = \sup(A)$, what can you say about A?