MA 509 – Tutorial 1 Solutions

(1) Suppose $\exists$ a rational number p/q, $q \neq 0$, $(p,q)=1$ such that $\frac{p^2}{q^2} = 12 \Rightarrow p^2 = 12q^2$

$\Rightarrow 4 \mid p^2$ and $3 \mid p^2$

$\Rightarrow 2 \mid p$ and $3 \mid p$

$\Rightarrow 6 \mid p$ so that $36 \mid p^2$

$$\Downarrow$$
$$36 \mid 12q^2$$
$$\Downarrow$$
$$3 \mid q^2$$
$$\Downarrow$$
$$3 \mid q.$$

$\Rightarrow (p,q) \geq 3$.

This contradicts the fact that $(p,q)=1$.
This proves the result. $\boxtimes$

(2) (a) 7 (b) $\pi + 1$ (c) π

(3) (a) $y = 1 \cdot y = \left(\frac{1}{x} \cdot x\right) \cdot y = \frac{1}{x} \cdot (xy) = \frac{1}{x} \cdot (xz)$
$= \left(\frac{1}{x} \cdot x\right) z = 1 \cdot z = z.$

(b) Let $z = 1$ in (a)

(c) Let $z = 1/x$ in (a)

(d) First, note that $\frac{1}{x} \cdot x = 1$.
Now replace x by $1/x$ in (c). Then it gives
$\frac{1}{x} \cdot y = 1 \Rightarrow y = \frac{1}{(1/x)}$. —— (*)

But using (a), $\frac{1}{x} \cdot y = \frac{1}{x} \cdot x \Rightarrow y = x$ —— (**)
From (*) & (**), $\frac{1}{(1/x)} = x$. $\boxtimes$

(4) $A \neq \phi$, $A \subseteq \mathbb{R}$. A is bounded below.

Let ~~A~~y be a lower bound of A.

$\Rightarrow y \leq x \quad \forall x \in A$

$\Rightarrow -y \geqslant -x \quad \forall x \in A$

Hence $-y$ is an upper bound of $-A = \{-x : x \in A\}$

By lub property, lub$(-A)$ (or $\sup(-A)$) exists in $\mathbb{R}$, say z.

Then $z \leq -y$

$\Rightarrow -z \geqslant y$

Note that z is an u.b. of $-A \Rightarrow z \geqslant -x \quad \forall -x \in -A \Rightarrow -z \leq x \quad \forall x \in A$.

$\Rightarrow -z$ is a lower bound of A.

Since y was any lower bound of A, this means

$-z = \inf(A)$ (greatest lower bound of A)

Thus $\inf(A) = -\sup(-A)$. $\boxtimes$

(5) $A \neq \phi$, $A \subseteq S$; S is an ordered set.

By lub property, $\inf(A)$ and $\sup(A)$ exist in S.

~~Let~~ We know ~~that~~ $\inf(A) \leq x \leq \sup(A)$

$\forall x \in A$.

But $\inf(A) = \sup(A)$.

Hence A ~~is~~ consists of a singleton set. $\boxtimes$