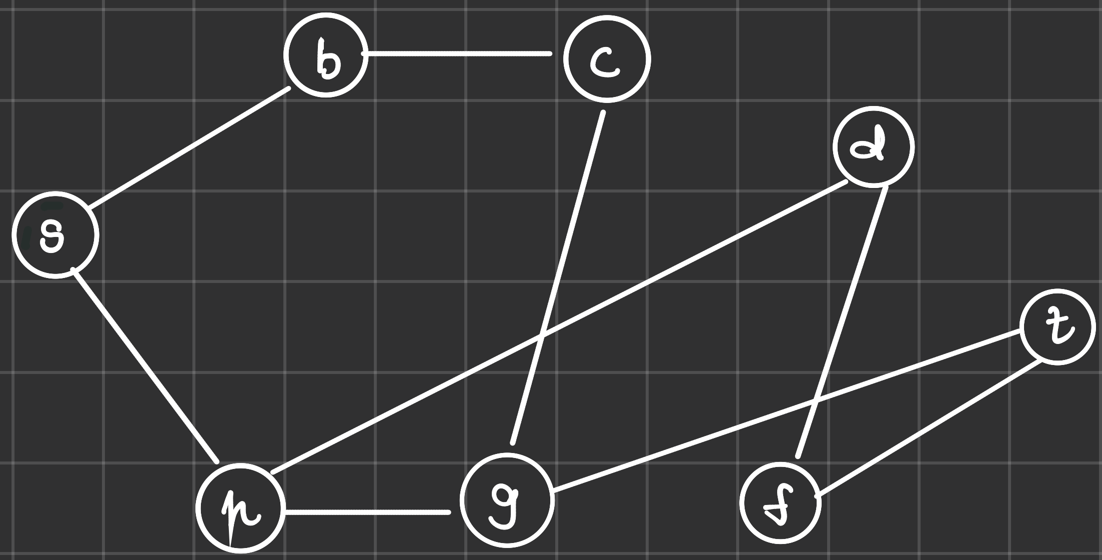
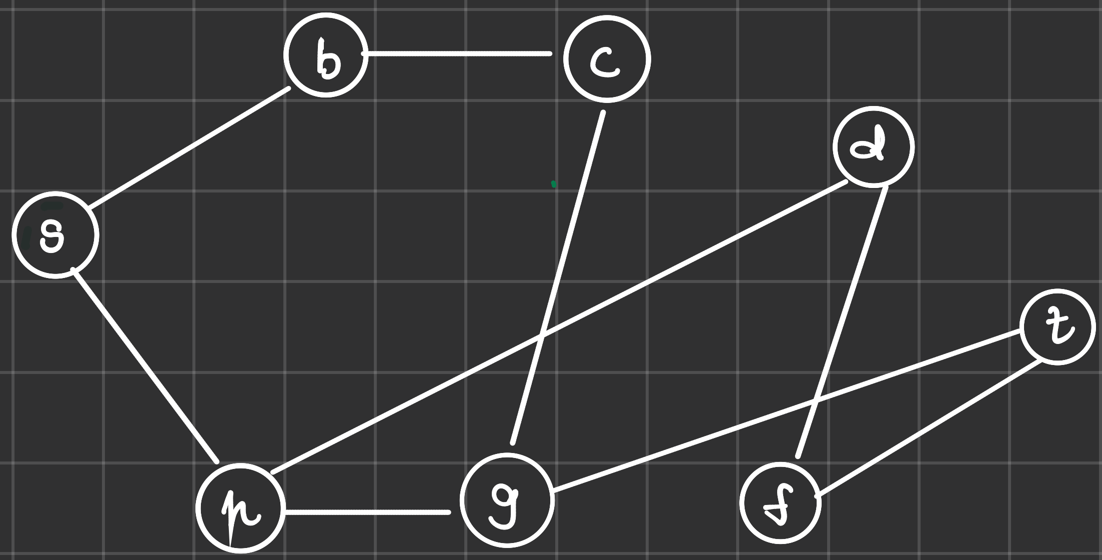
Graphs



A Graph has a set of nodes or vertices and edges between them.

Graphs



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where V is a set of nodes
and $E \subseteq V \times V$

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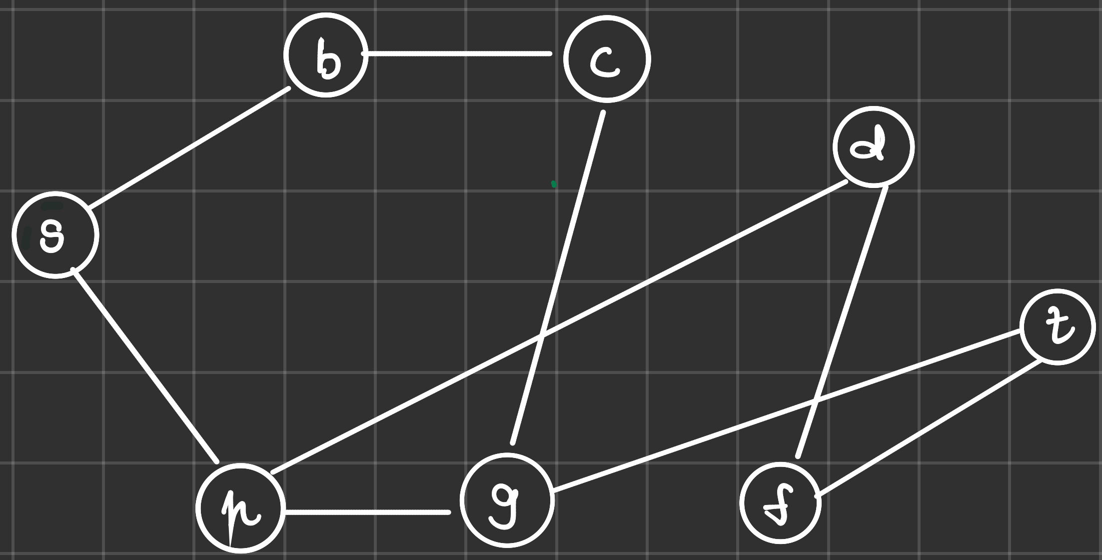

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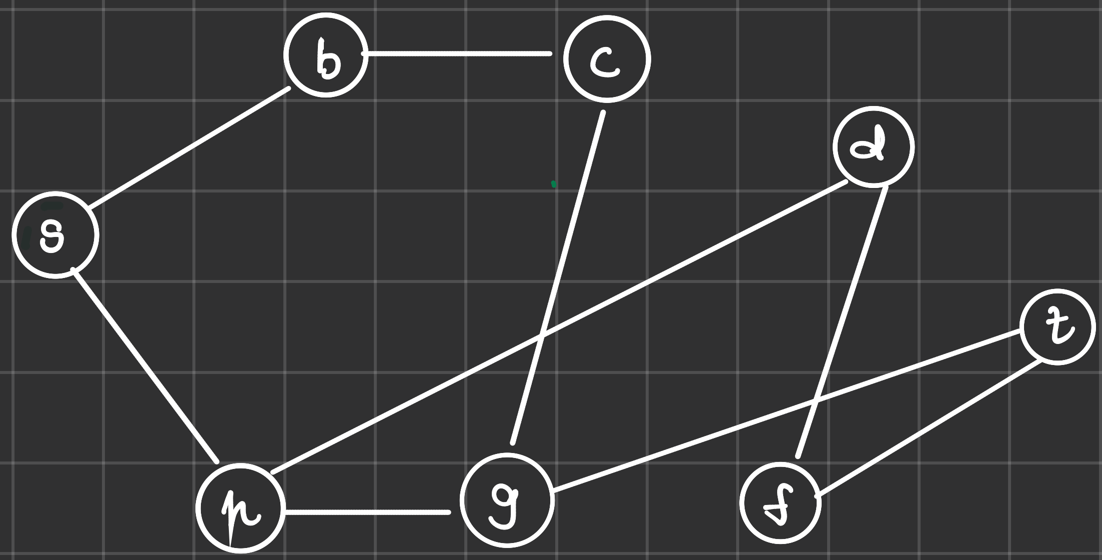

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$G(\{s, b, c, d, f, g, h, t\}, \{(s,b), (b,c), (s,h), (h,g), (c,g), (h,d), (d,f), (g,t), (f,t)\}$

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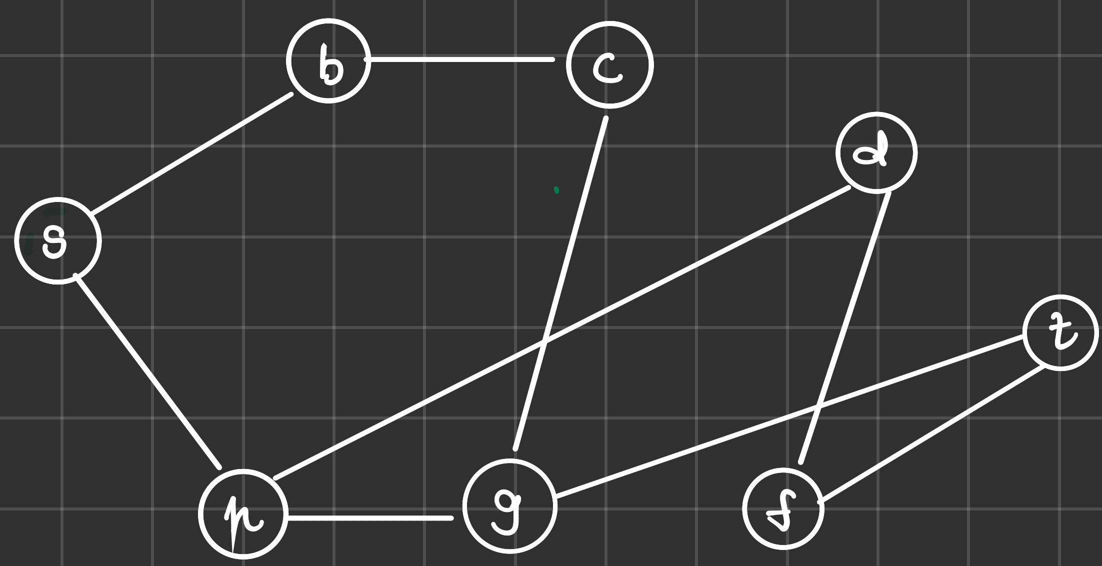

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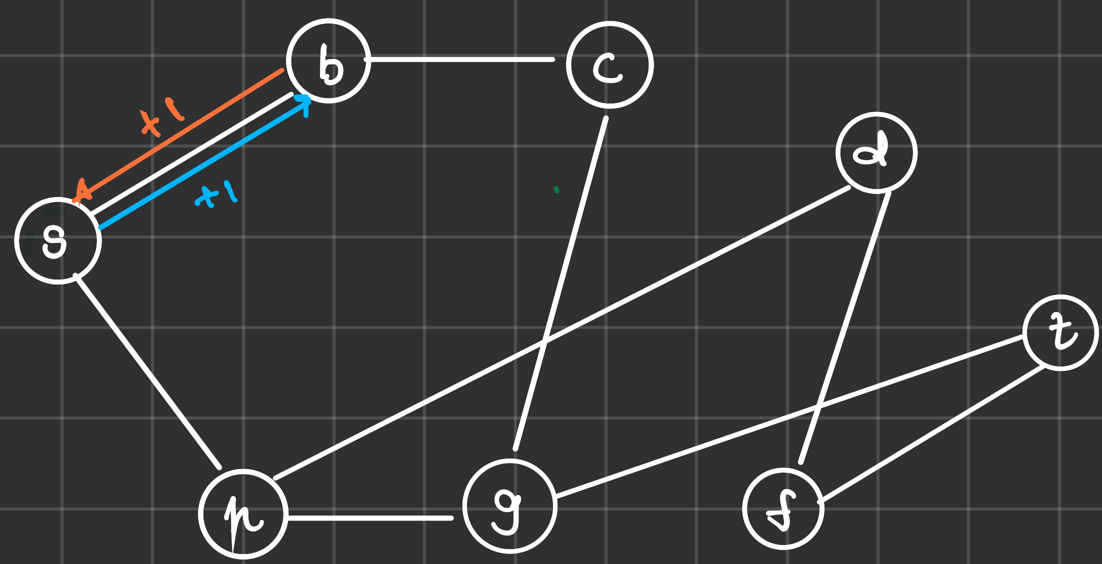

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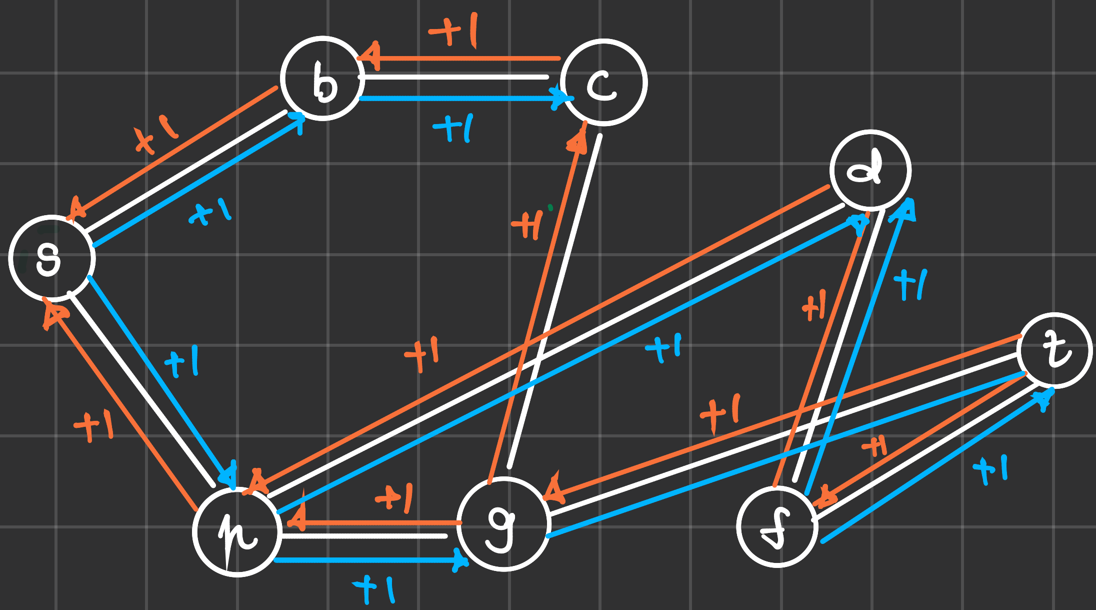

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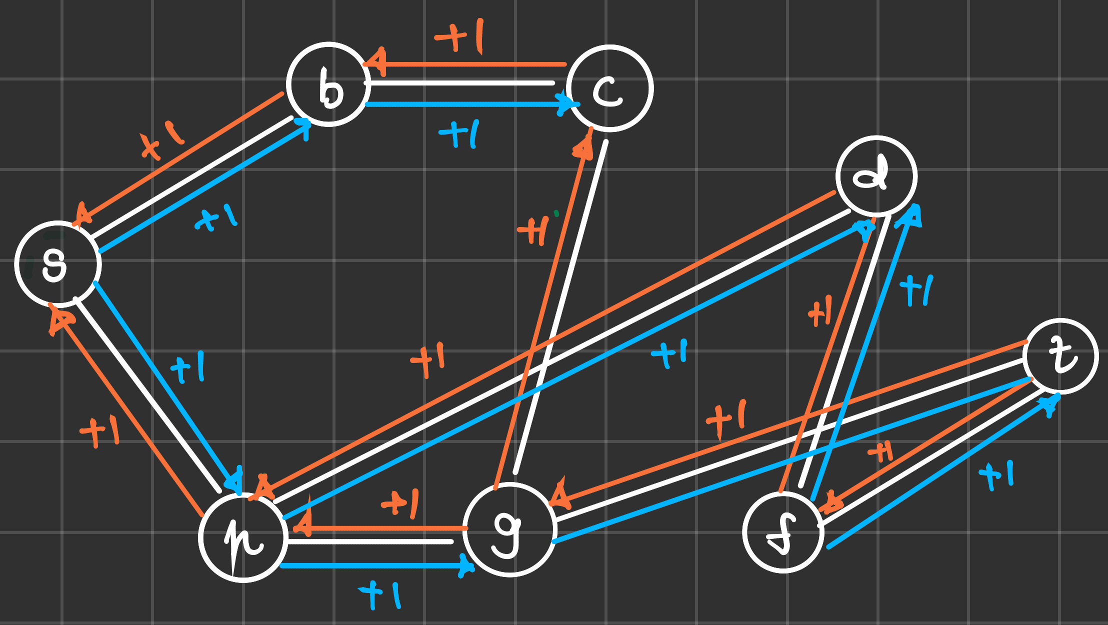
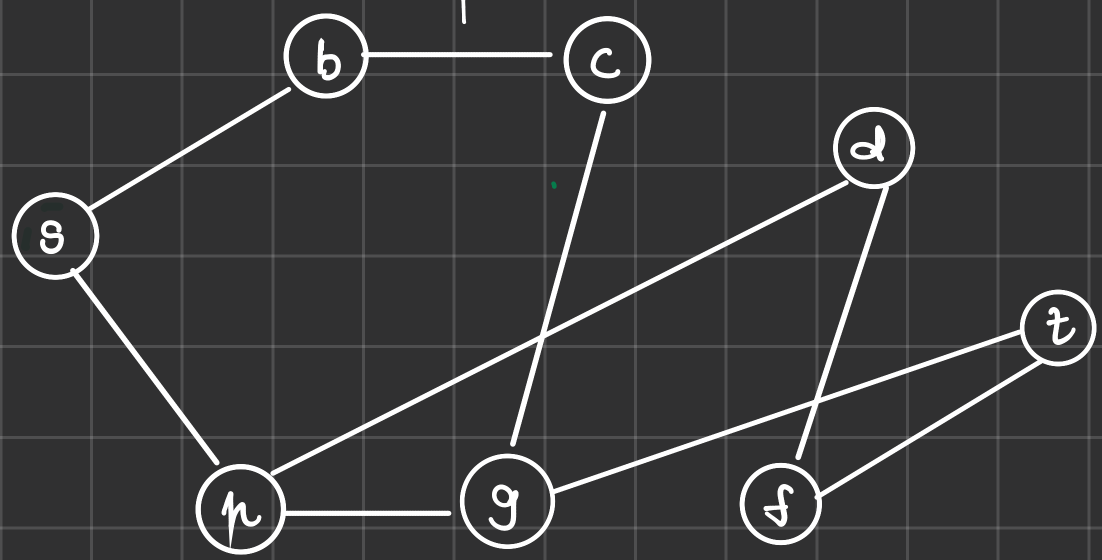


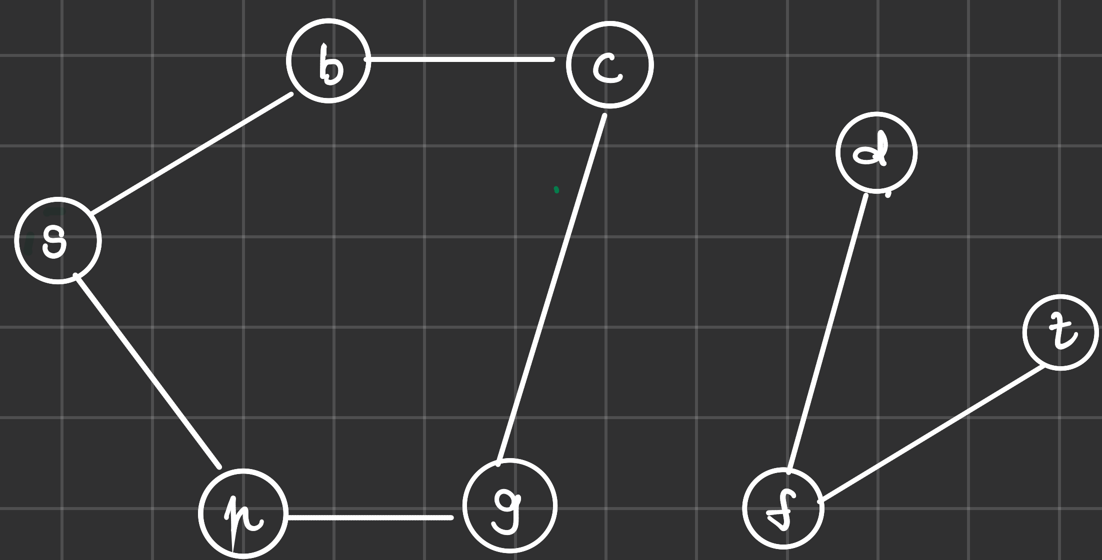
In $\sum_{v \in V} d_v$ each edge is added exactly twice.

$$\Rightarrow \sum_{v \in V} d(v) = 2m.$$

Connected Graph.



Disconnected Graph



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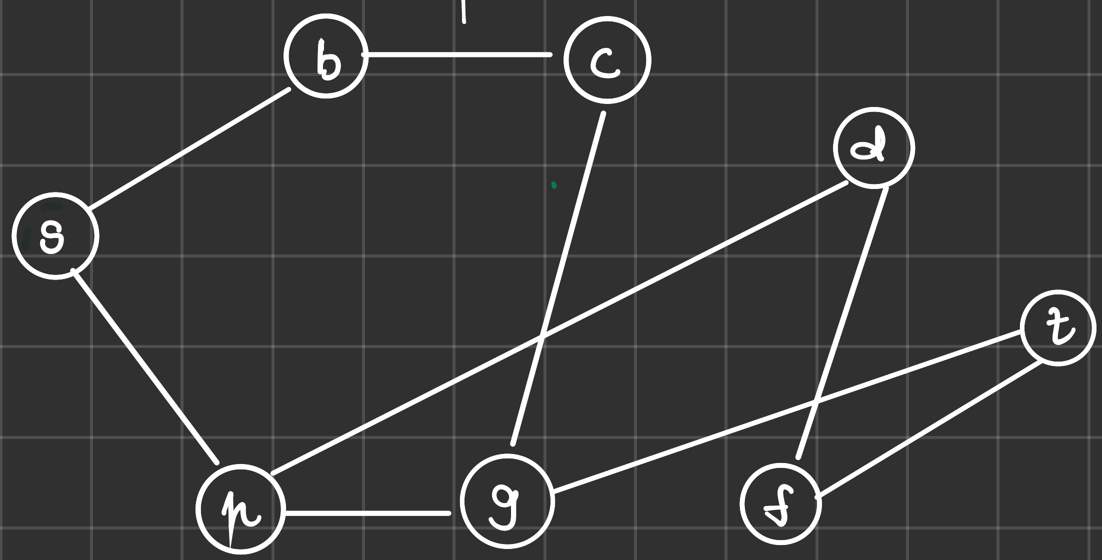

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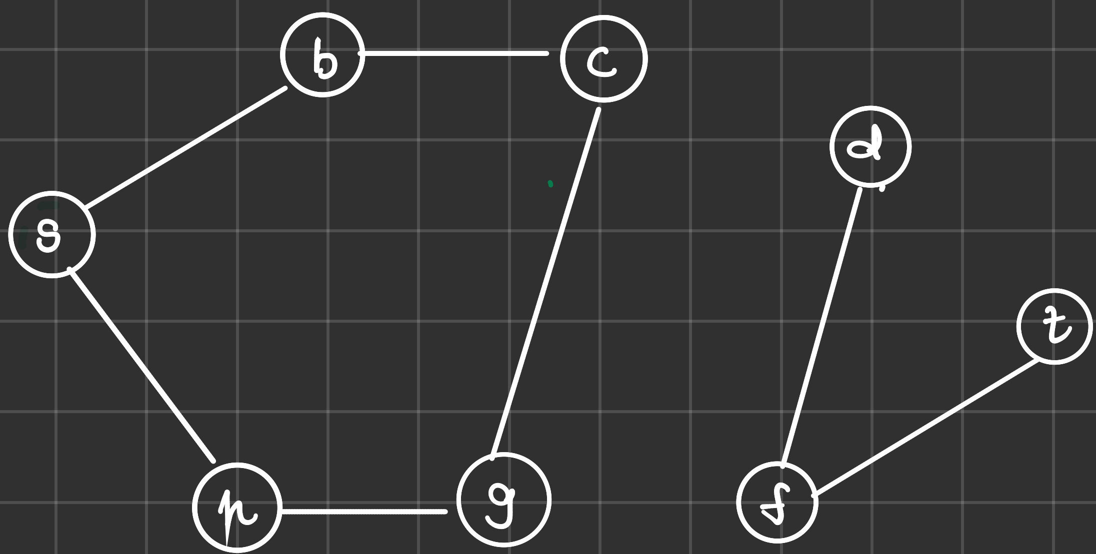

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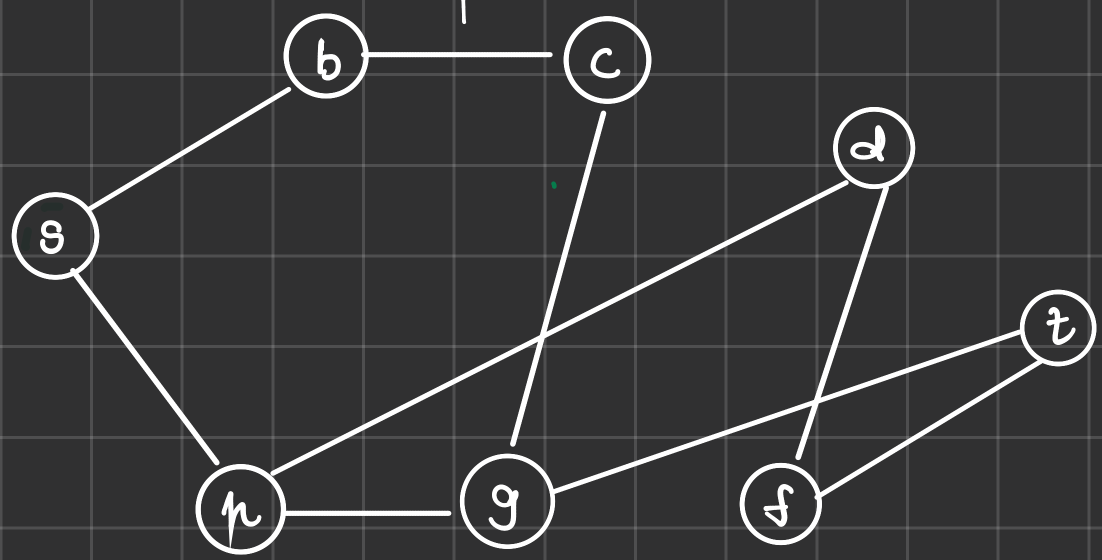

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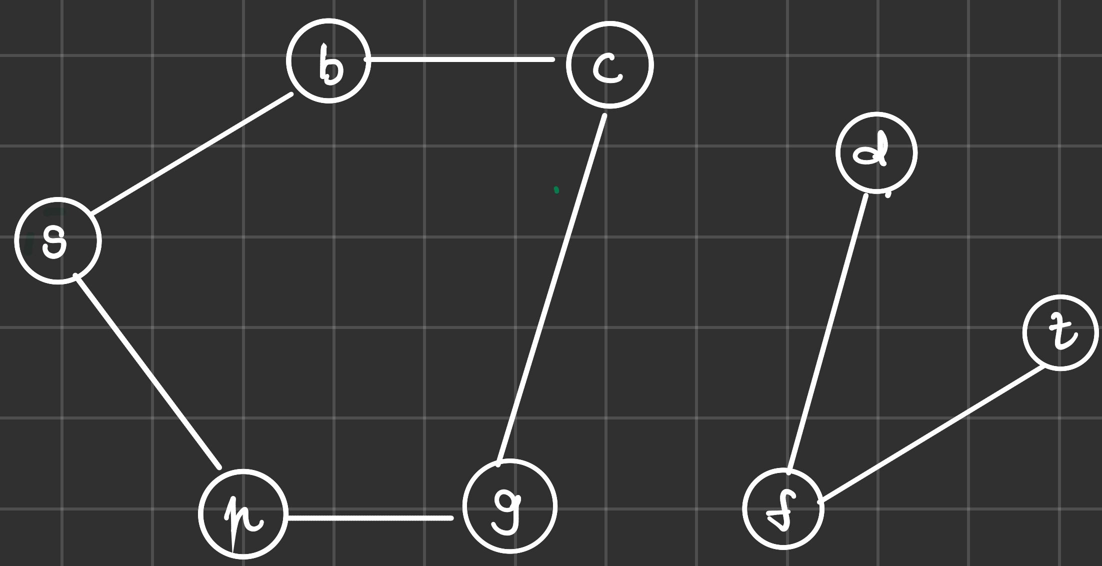

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A: $\frac{n \cdot (n-1)}{2}$

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$\uparrow$

This graph is also called a tree.

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Assume that the statement is true for all values in range $[1, n-1]$

Prove the statement when there are n vertices

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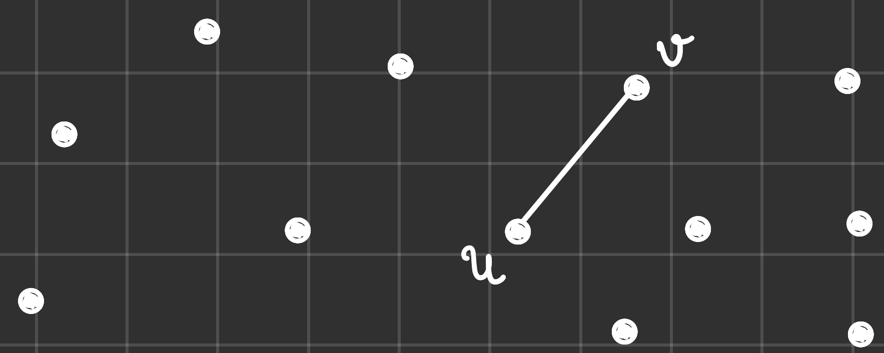
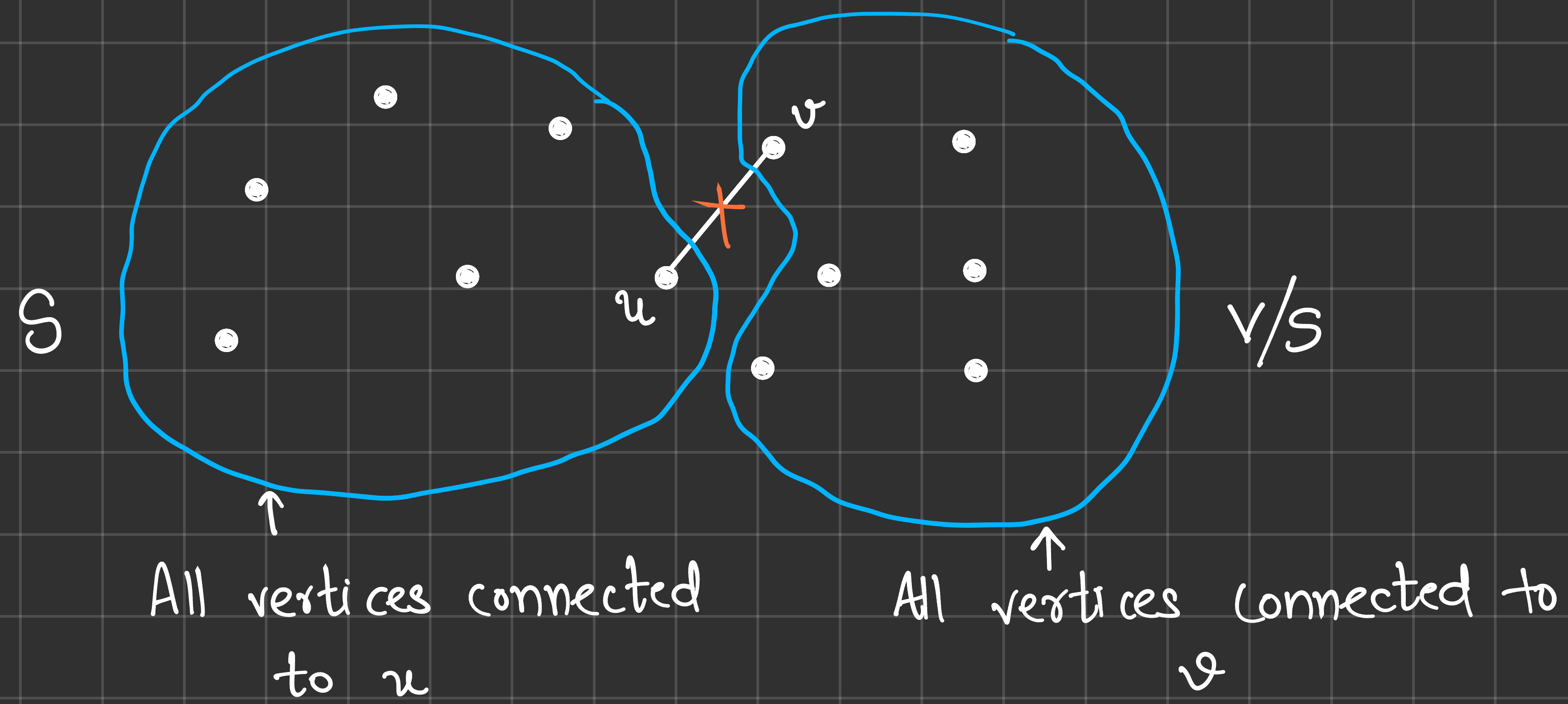
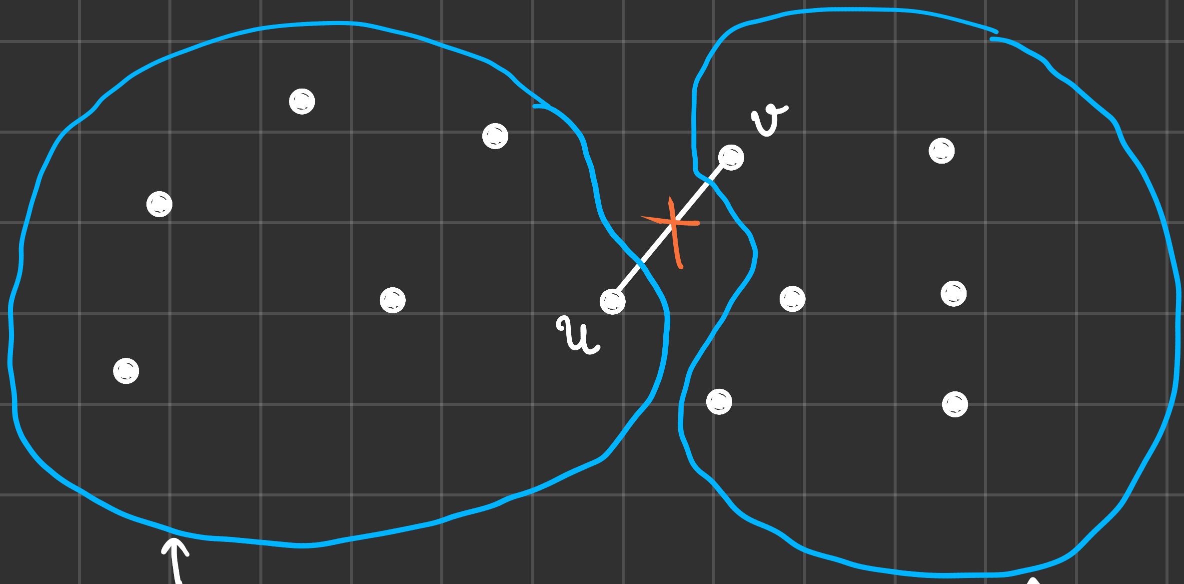
Prove the statement when there are n vertices

Assume that there exists a tree on n vertices





We have divided the graph into two connected components.



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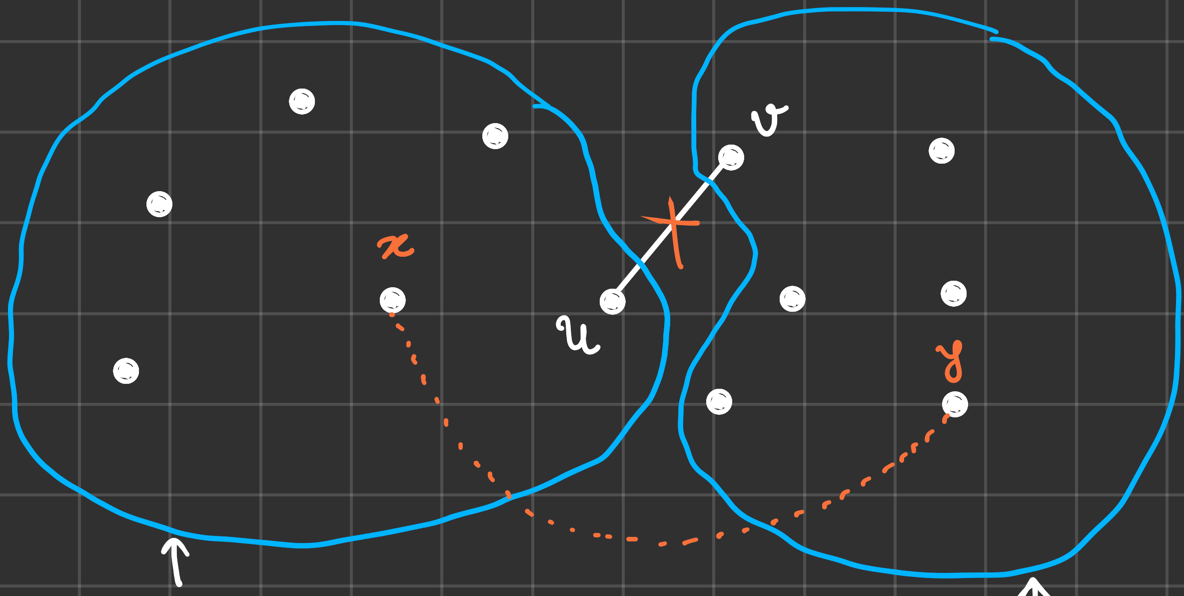

All vertices connected to u

All vertices connected to v

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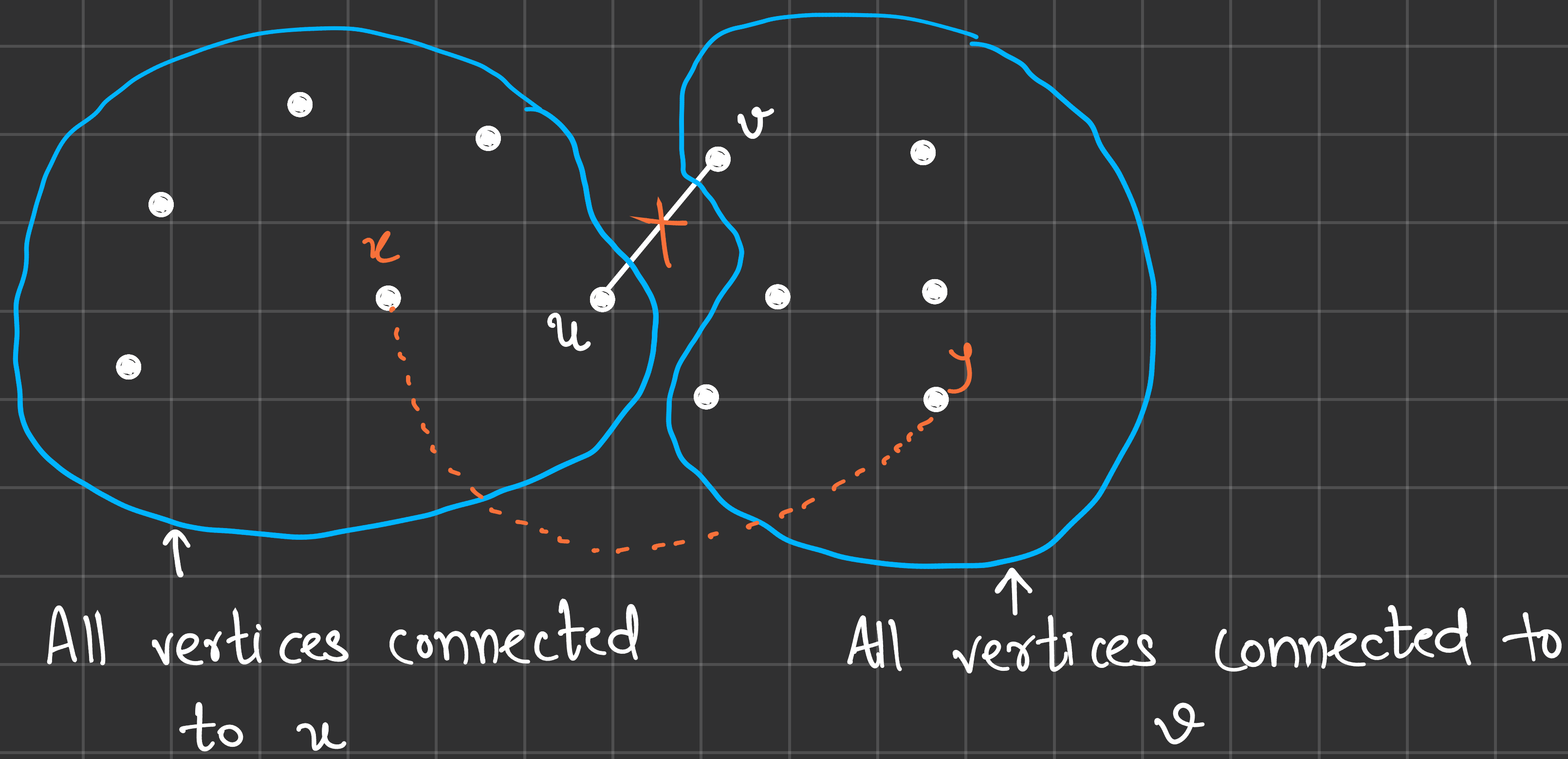
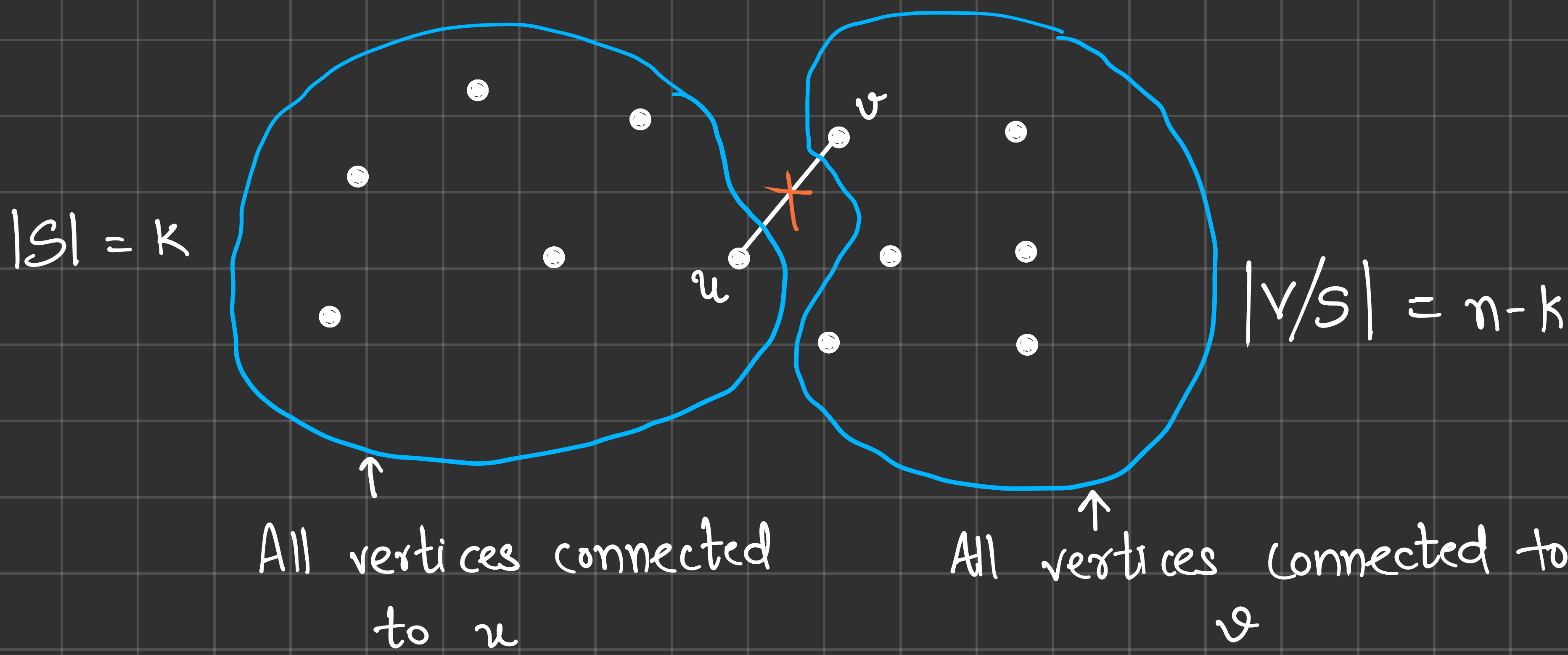


We have divided the graph into two connected components. or have we ?!

$\Rightarrow$ $u \rightsquigarrow x + xy + y \rightsquigarrow v + vu$ is a cycle.
But we started out with a tree

$\Rightarrow$ Edge xy cannot exist



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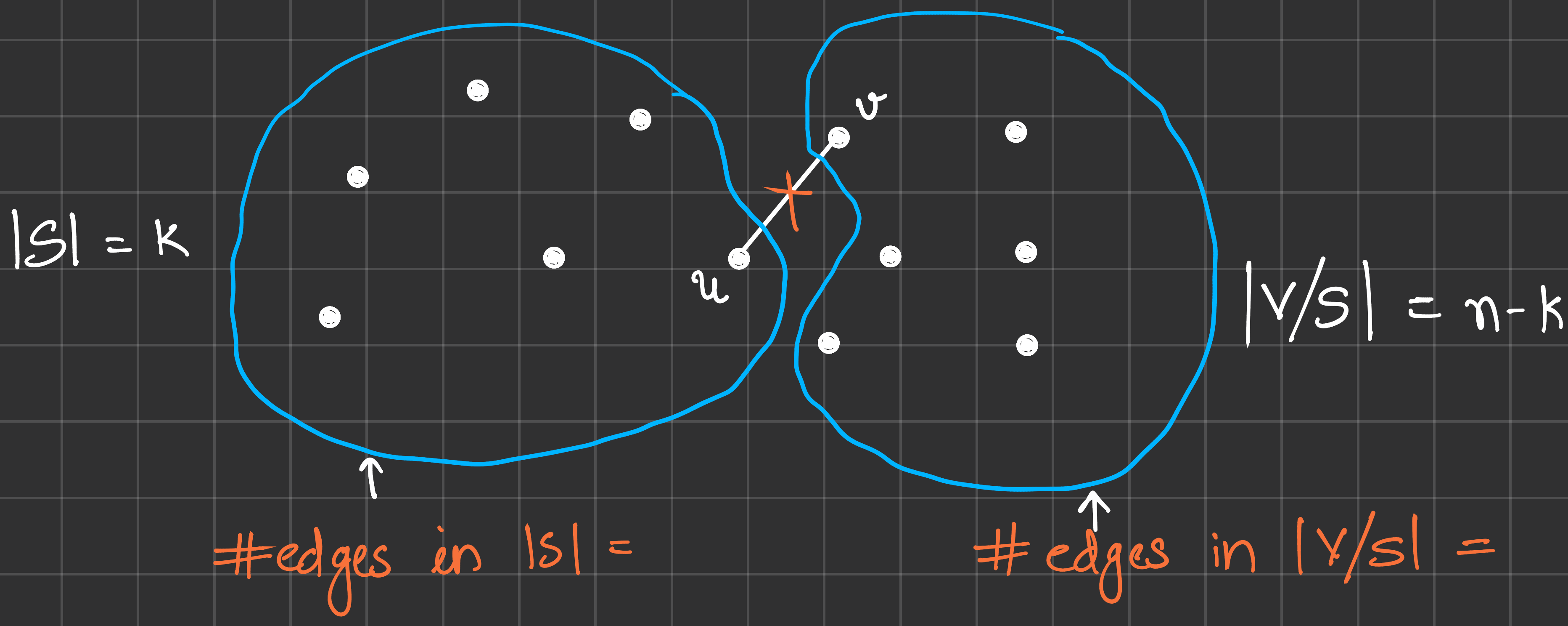

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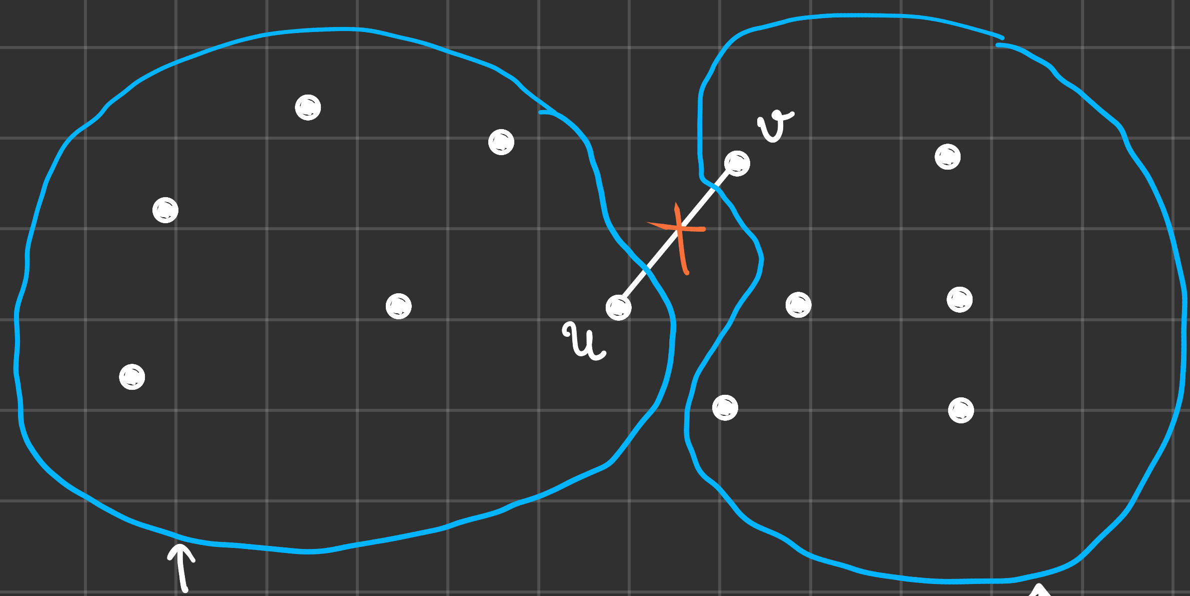
$|S| = K$



$|V/S| = n-k$

$\#$edges in $|S| = K-1$

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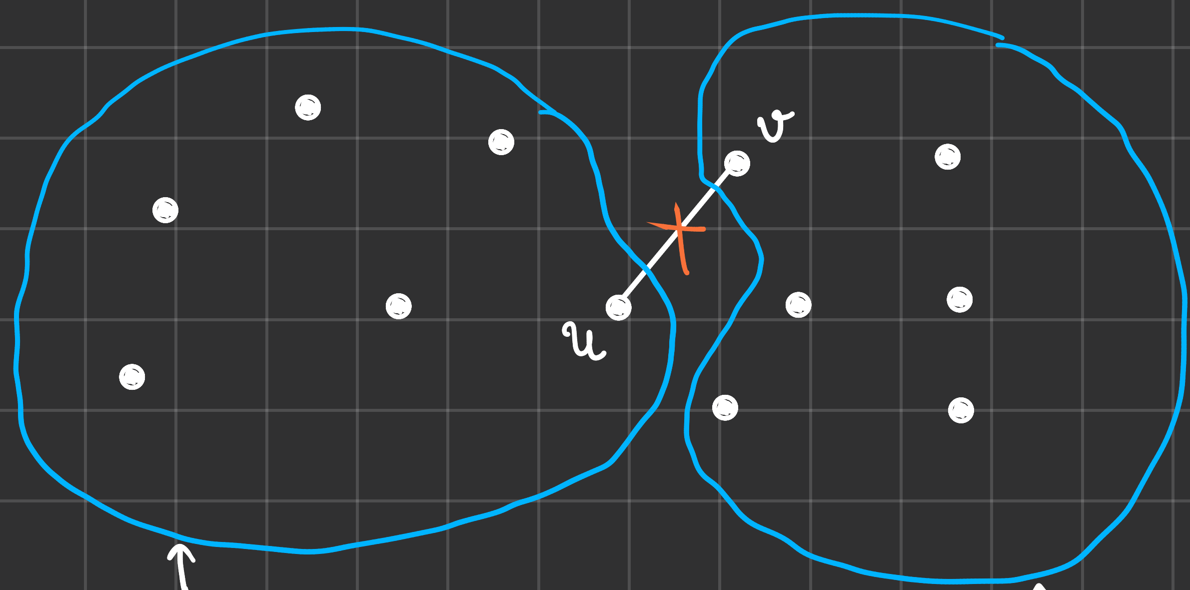

$|V/S| = n-k$

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We have divided the graph into two connected components.

#edges in our tree $= k-1 + n-k-1 + 1$

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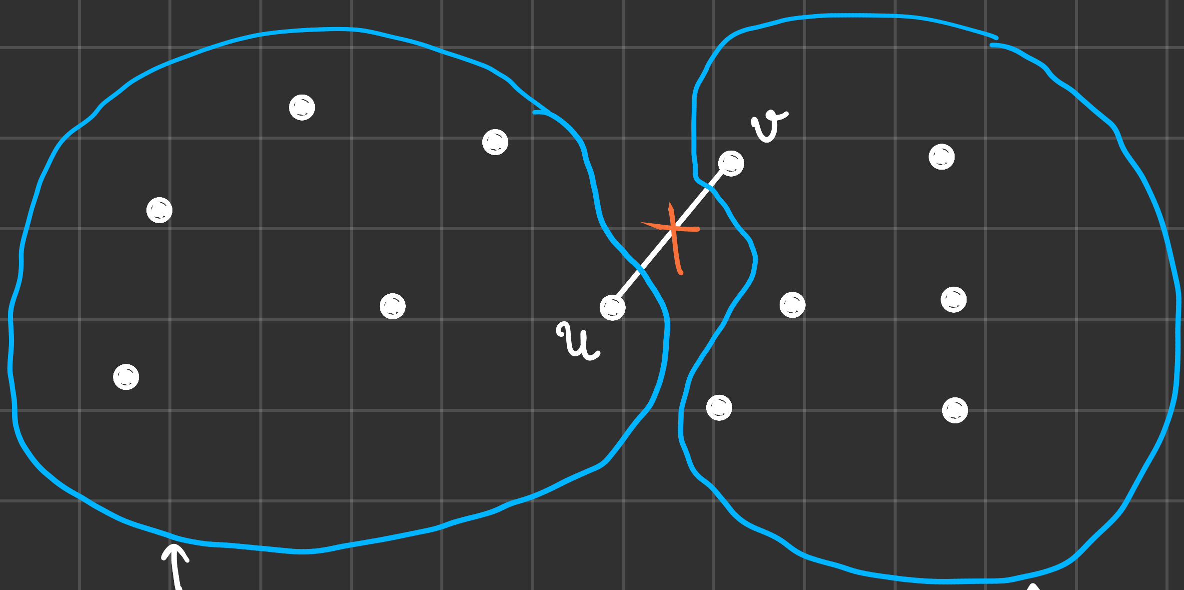

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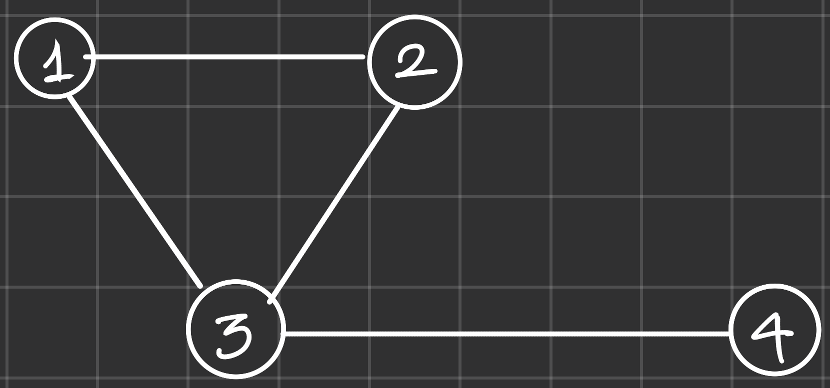
$$= n-1$$

Data structure for graph representation



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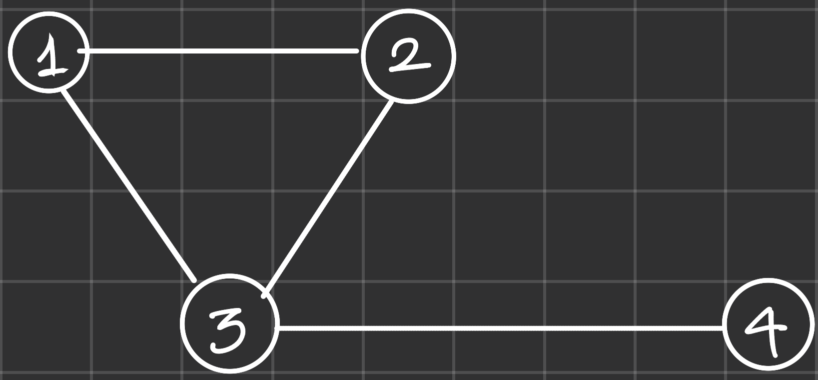
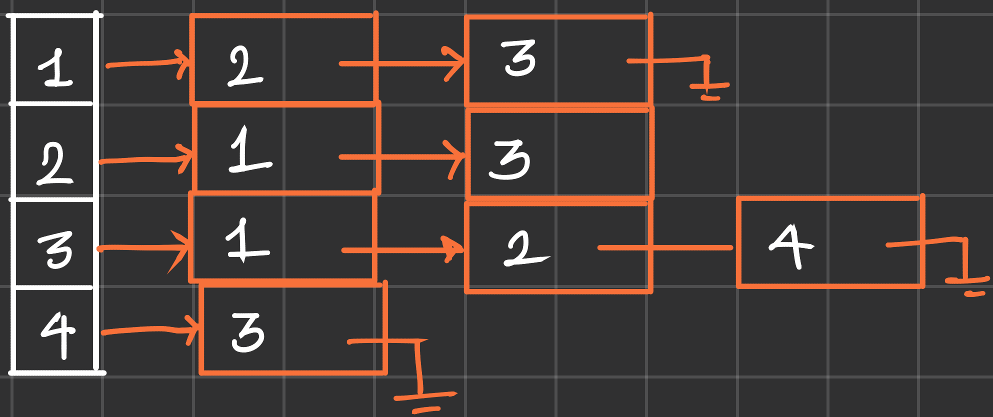


Adjacency list



Adjacency matrix

$$
\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 1 & 1 & 0 \\
2 & 1 & 0 & 1 & 0 \\
3 & 1 & 1 & 0 & 1 \\
4 & 0 & 0 & 1 & 0 \\
\hline
\end{array}
$$

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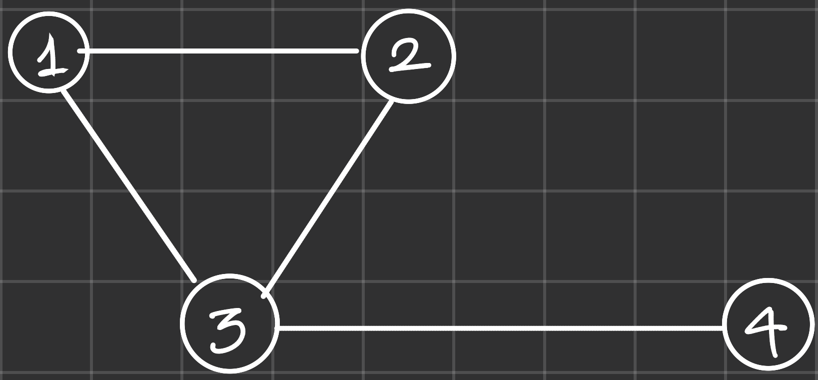

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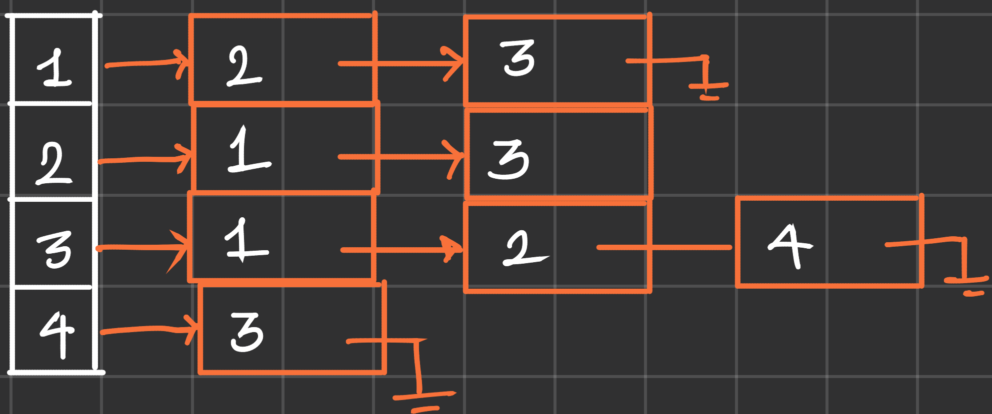

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Space:

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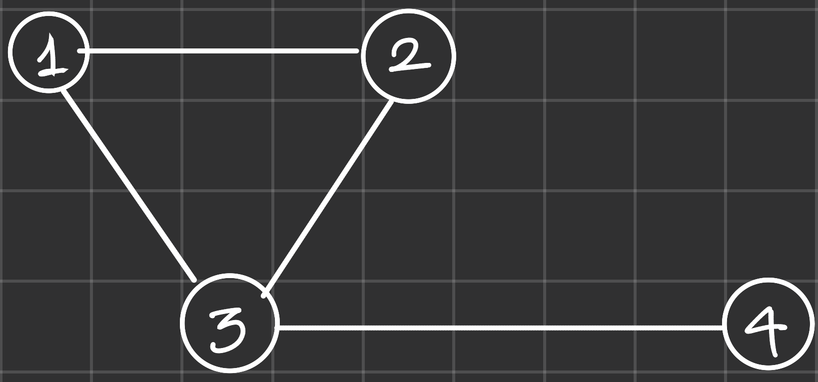

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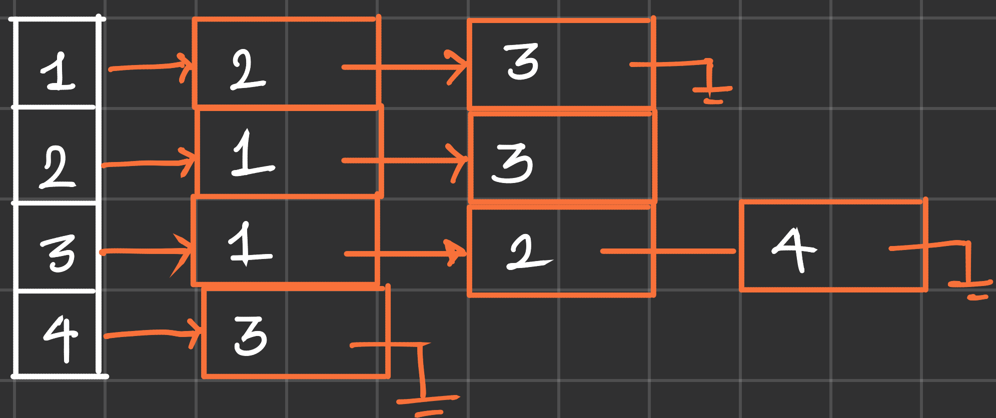

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Space: $O(m+n)$ $O(n^2)$

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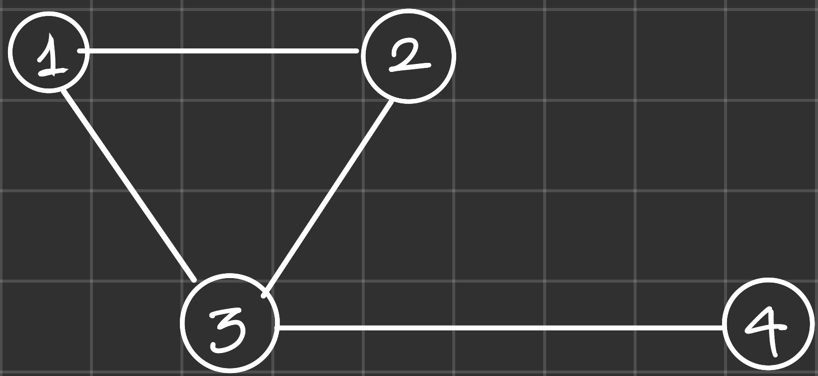

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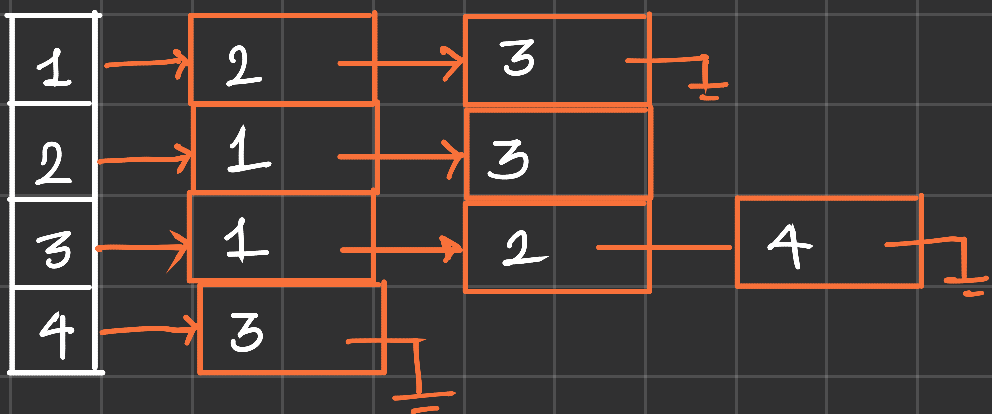

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$$

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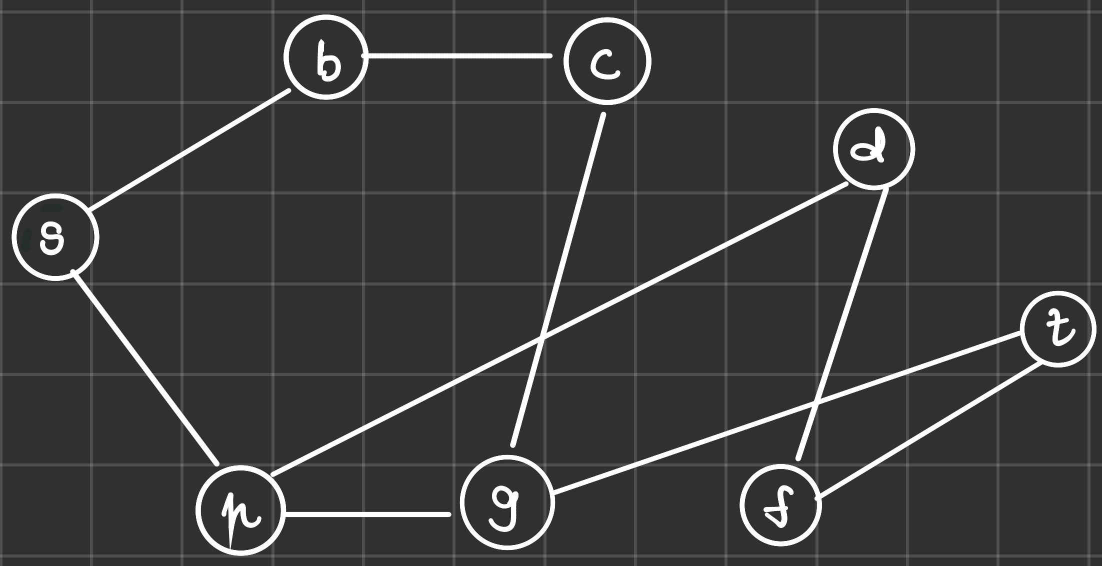
Unless stated otherwise, we will assume that the graph is given in adjacency form list.

Q: Given a graph G and two vertices s & t, find if t is reachable from s.



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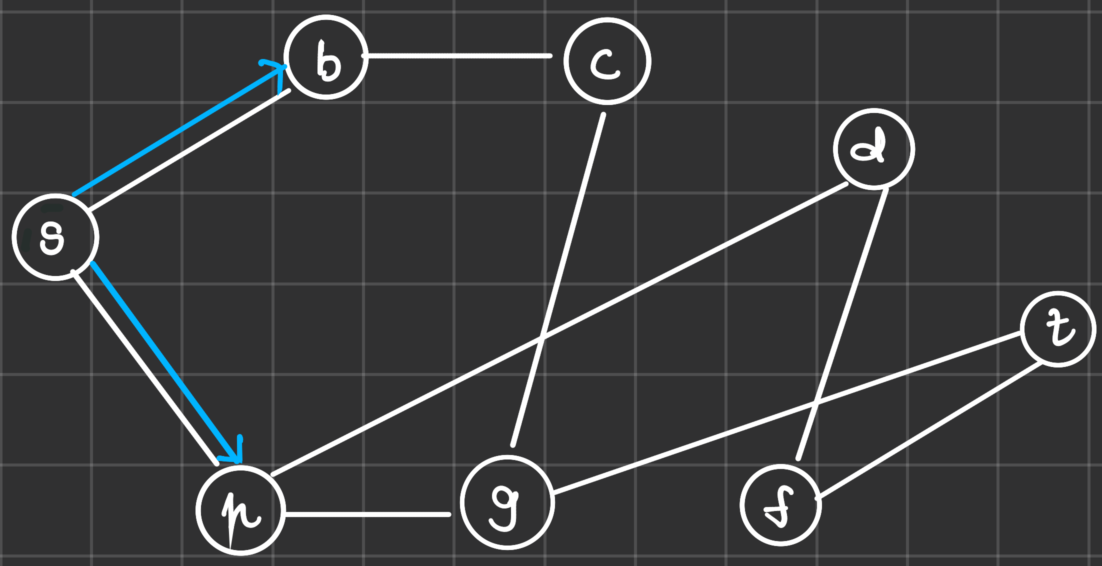

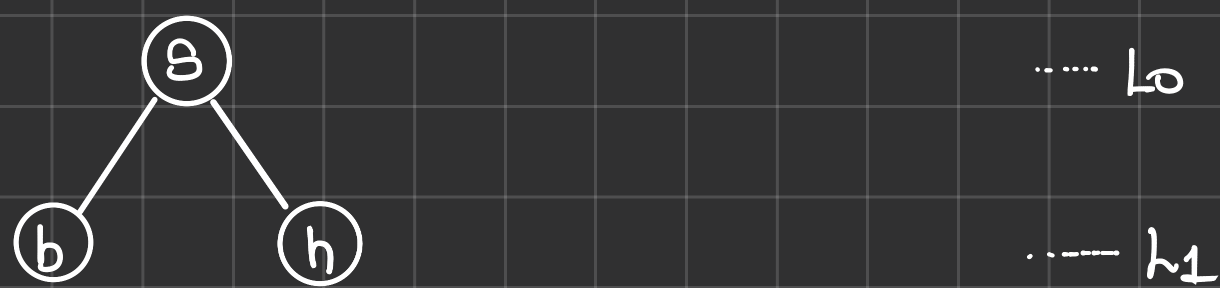

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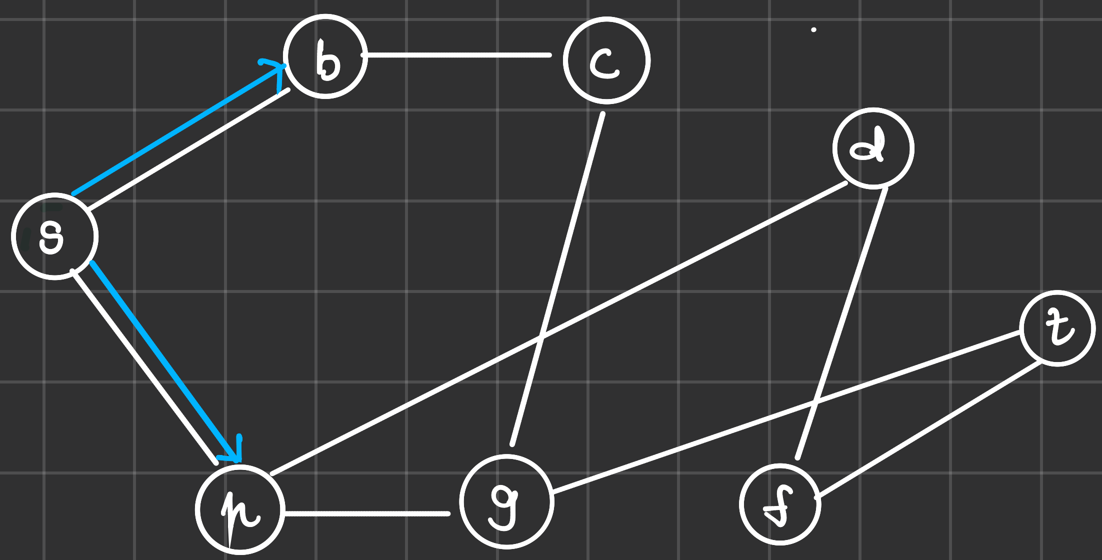

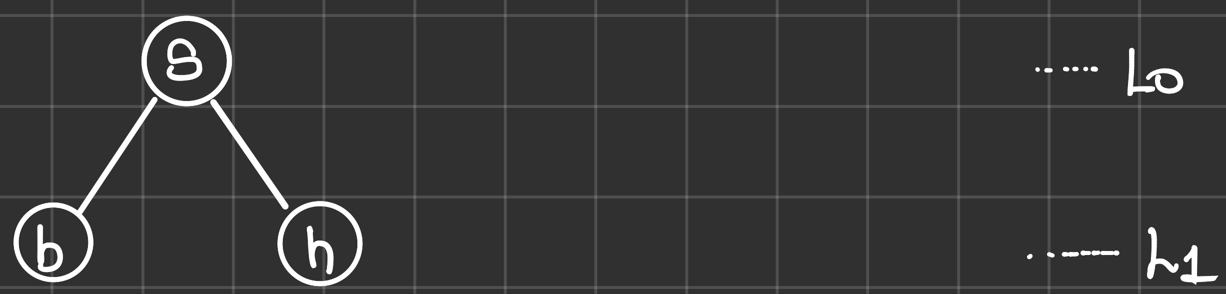

1) consider nodes in order
2) discover new vertices

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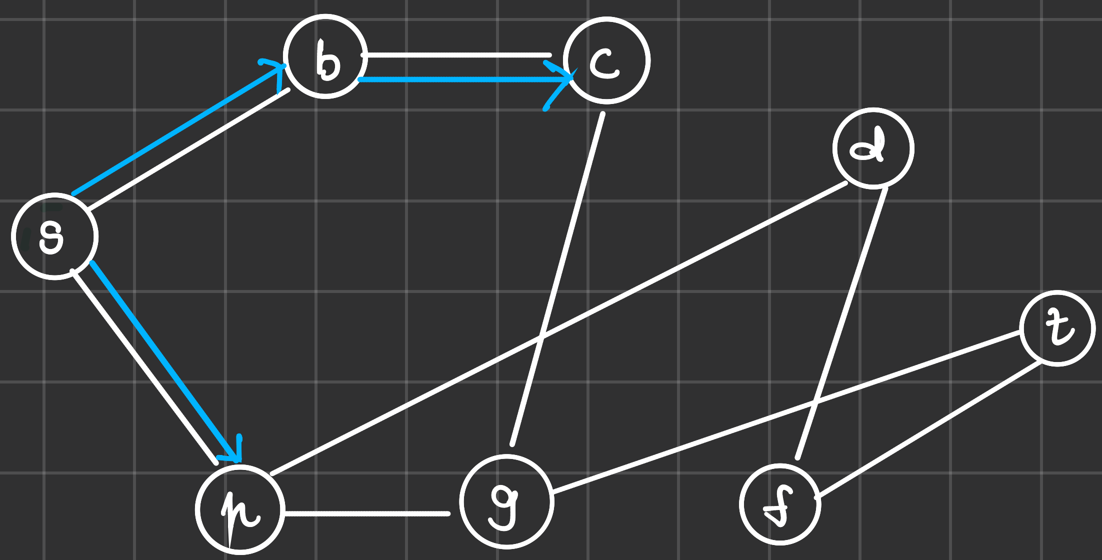

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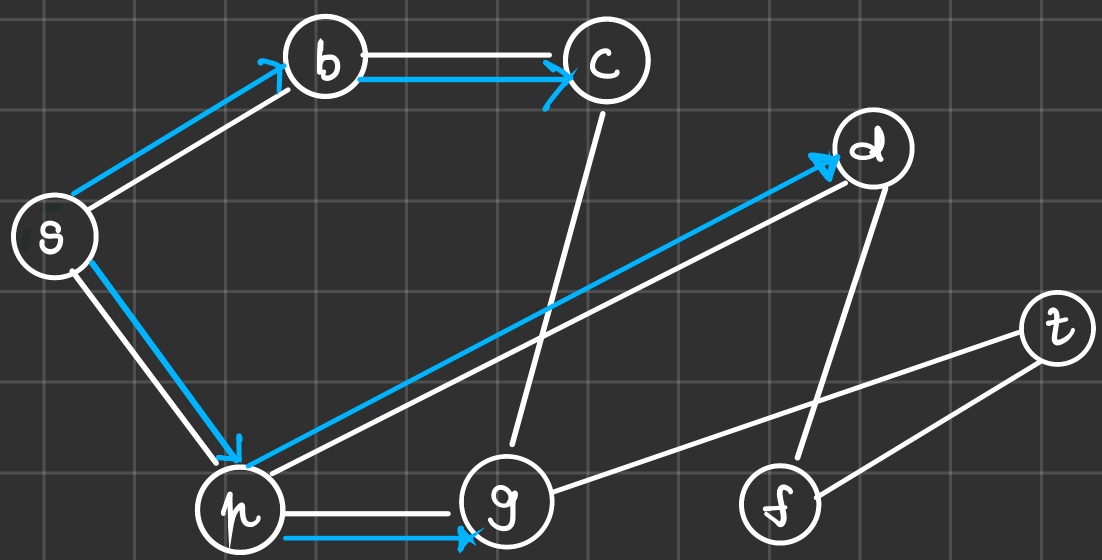

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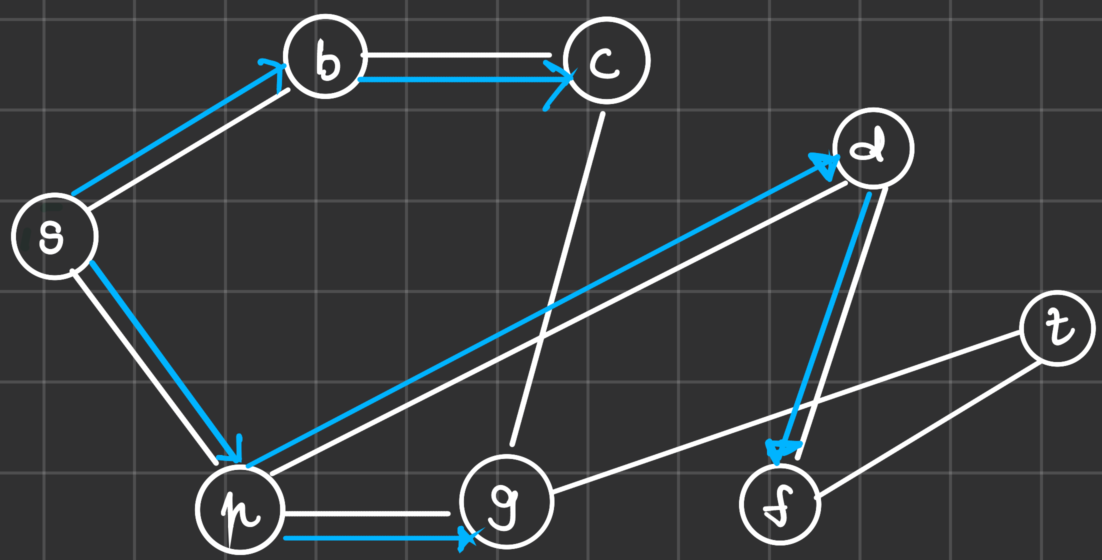

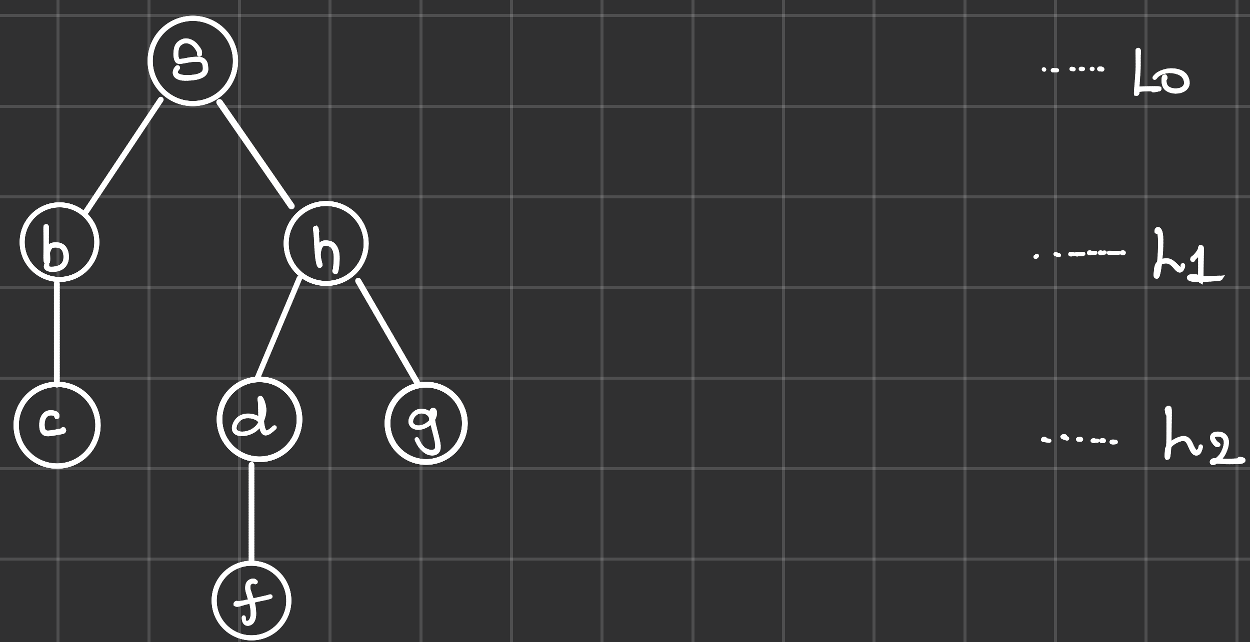

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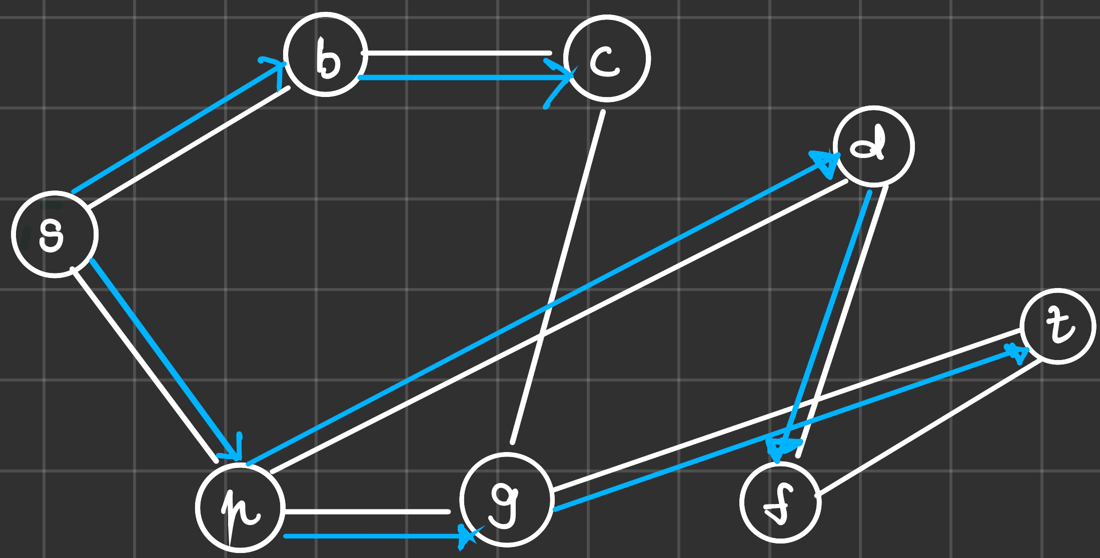

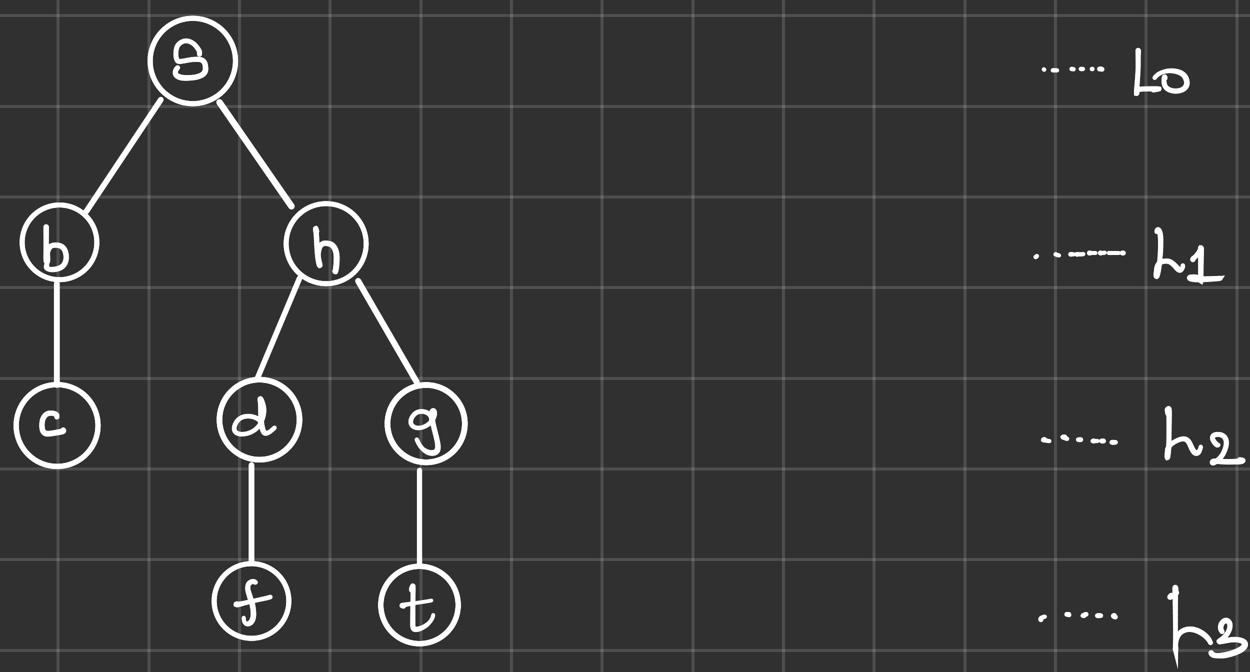

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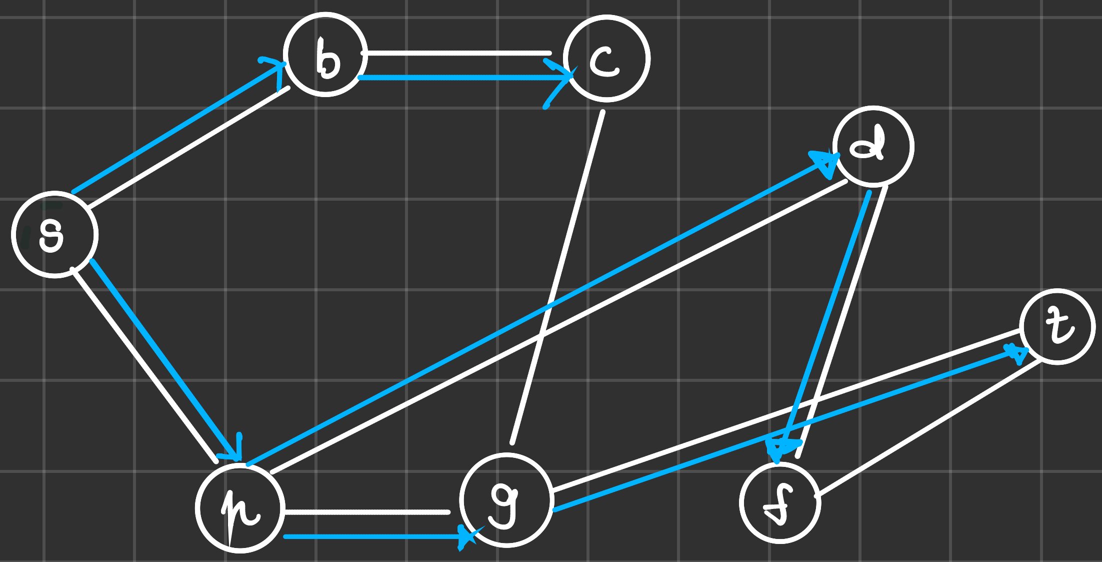
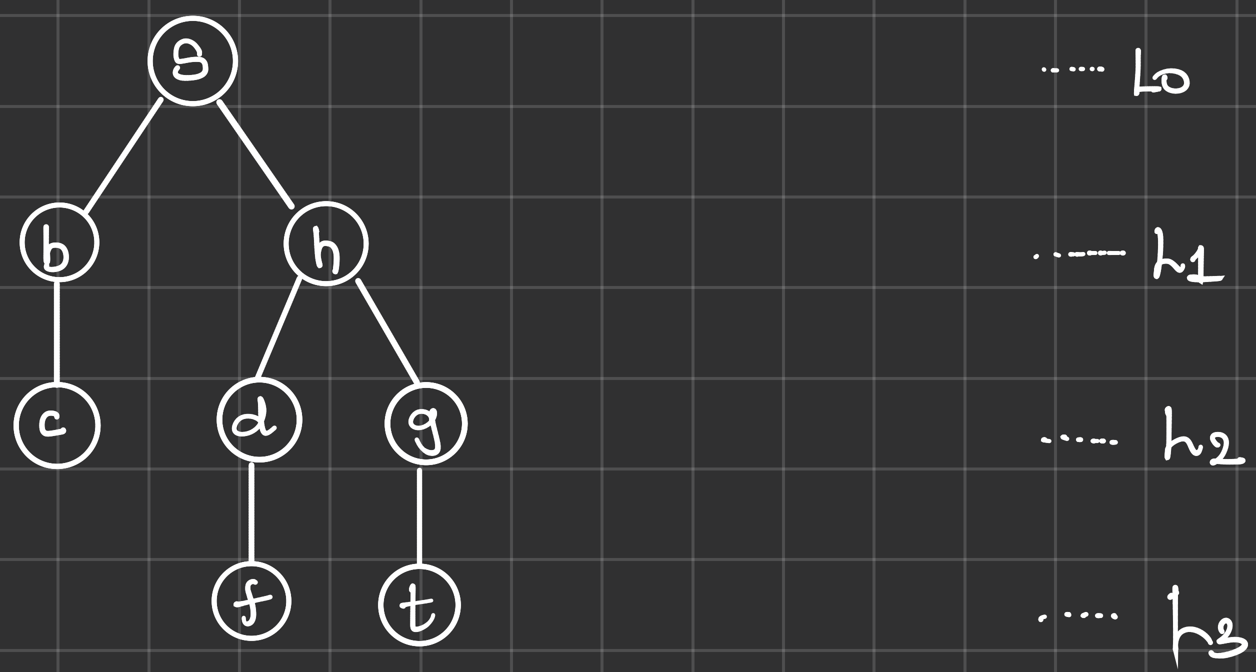


BFS tree



Tree Edges : all the edges in BFS tree (or all blue edges).

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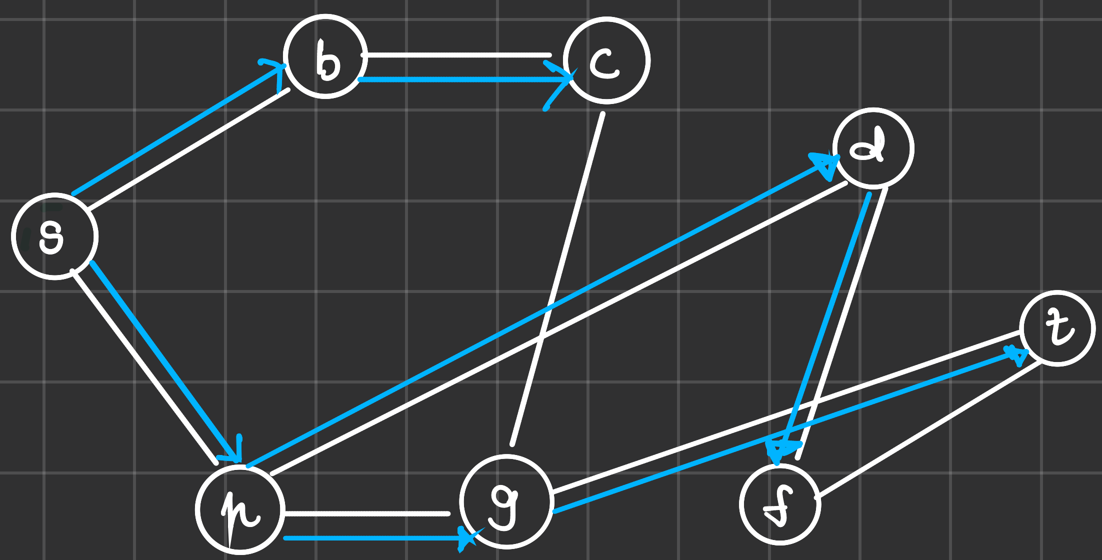

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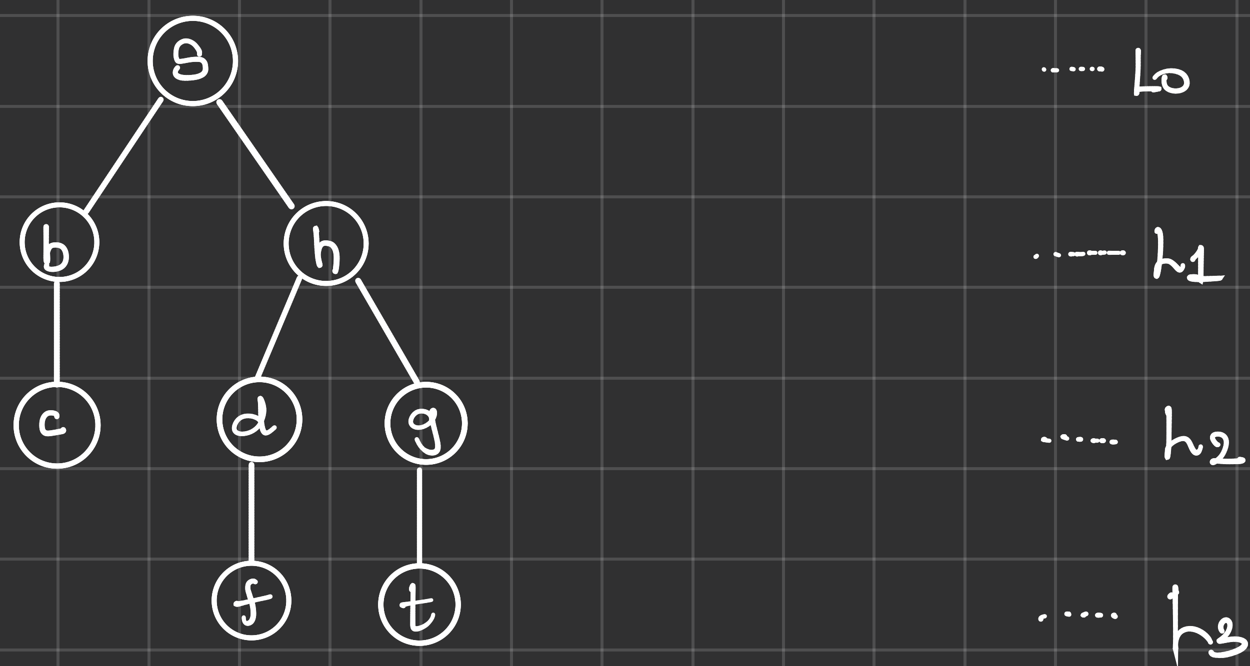

Tree Edges : all the edges in BFS tree (or all blue edges).

Non-tree edges: edges not present in the BFS tree

{ (c,g) , (f,t) }

<u>Observation</u>: If distance from s to x, $d_G(s,x) = \ell$, then x will be part of layer i:

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$\Rightarrow$ If s & t are connected, then $d(s,t) = c$

$\Rightarrow$ t will lie in the BFS-tree

Observation: If distance from s to x, $d_G(s,x) = l$, then x will be part of layer : L_l

$\Rightarrow$ If s & t are connected, then $d(s,t) = c$

$\Rightarrow$ t will lie in the BFS - tree

Some more properties of Breadth First Search.

Lemma: Let xy be an edge in the graph. Let x lie in layer L_i & y lie in layer L_j then $|i-j| \leq 1$.

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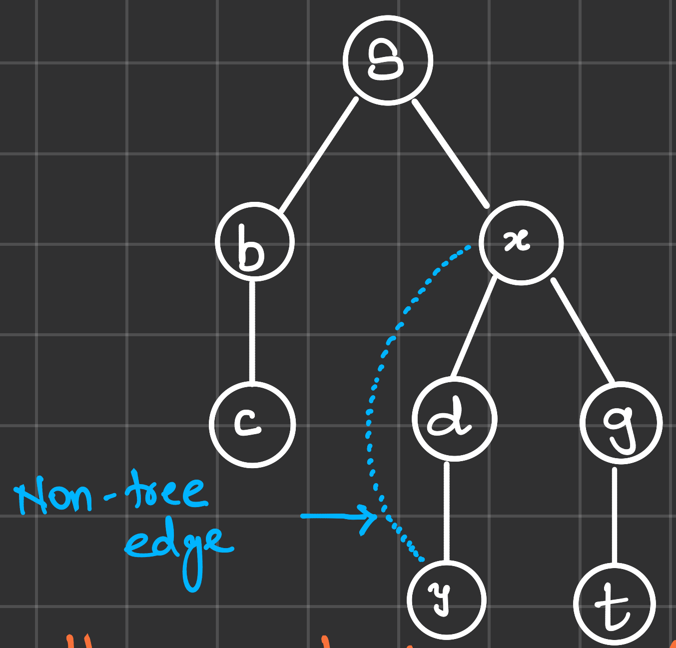

Why can this not happen !

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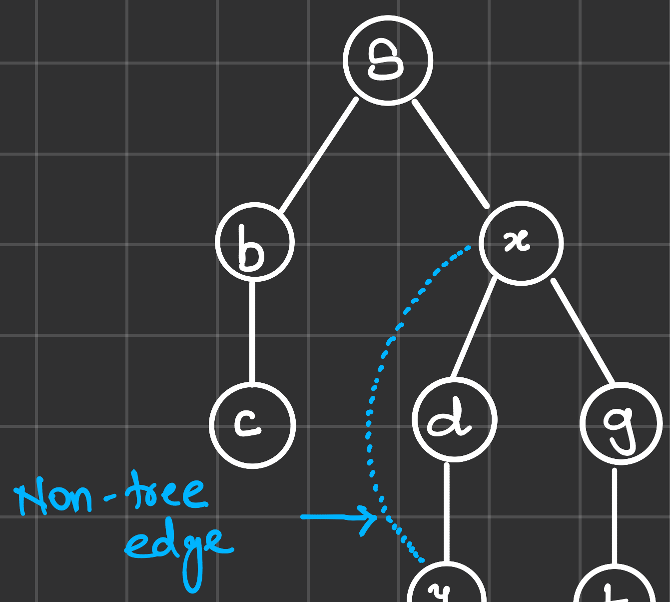
Some more properties of Breadth First Search.

Lemma: Let xy be an edge in the graph. Let x lie in layer L_i & y lie in layer L_j then $|i-j| \leq 1$.



When x is processed, it should have found y. So y should be the child of x.

Implementing BFS

```
BFS (s)
{   for each v ∈ V
    {
        discovered[v] ← false;
    }
    discovered[s] ← true;

    Q.enqueue(s)
    while (Q is not empty)
    {   v ← Q.deque();
        for each neighbor w of v
        {   if (discovered[w] = false)
            {


            }
        }
    }
}
```

Implementing BFS

```
BFS (s)
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    {
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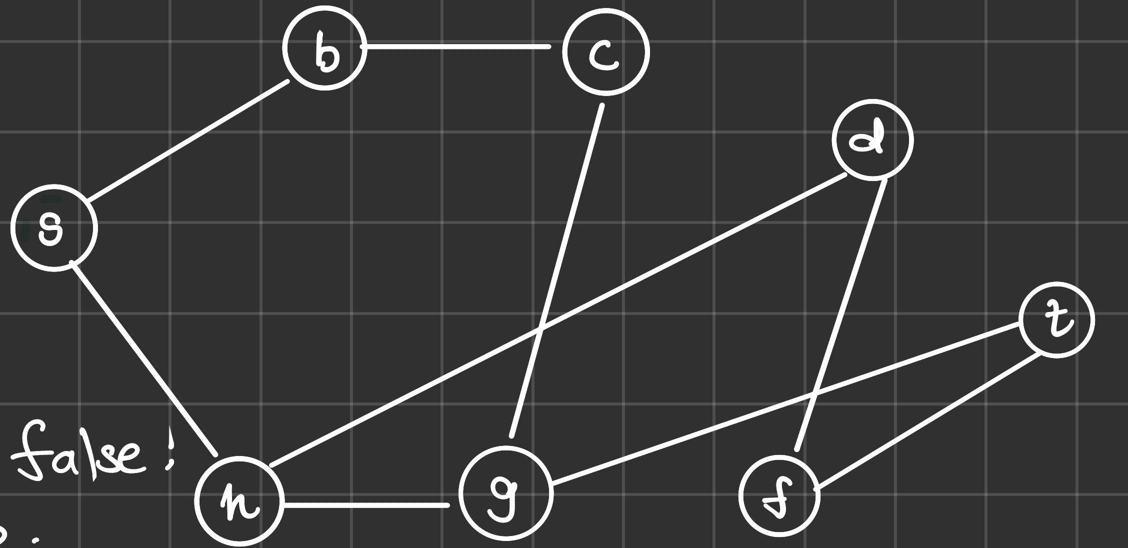
        {   if ( discovered [w] = false)
            {
                discovered [w] = true  ;   Q. enque(w) ;
                add  (v,w)  to  the  BFS  tree;
            }
        }
    }
}
```

Implementing BFS

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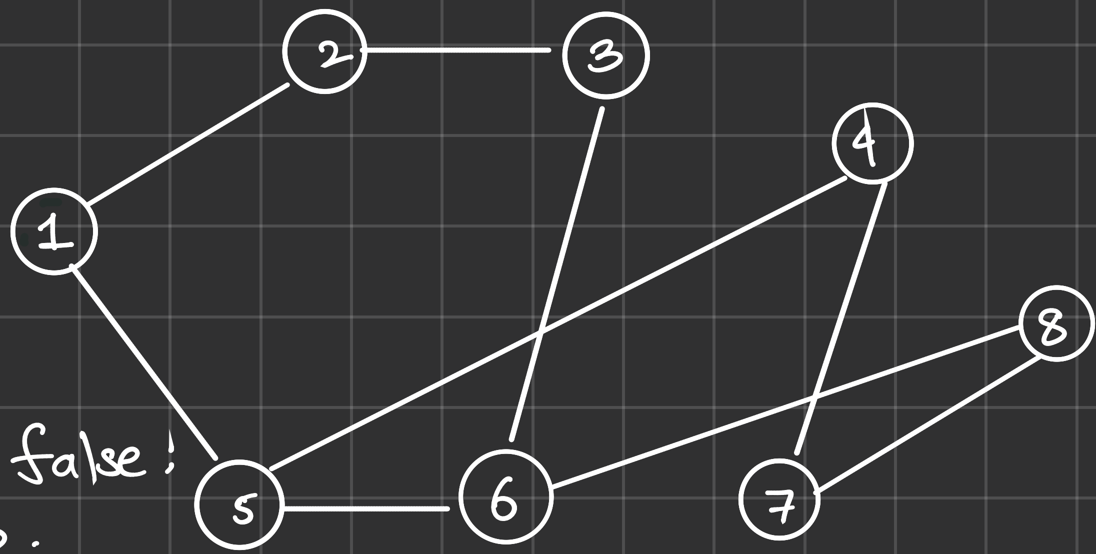
```

Implementing BFS

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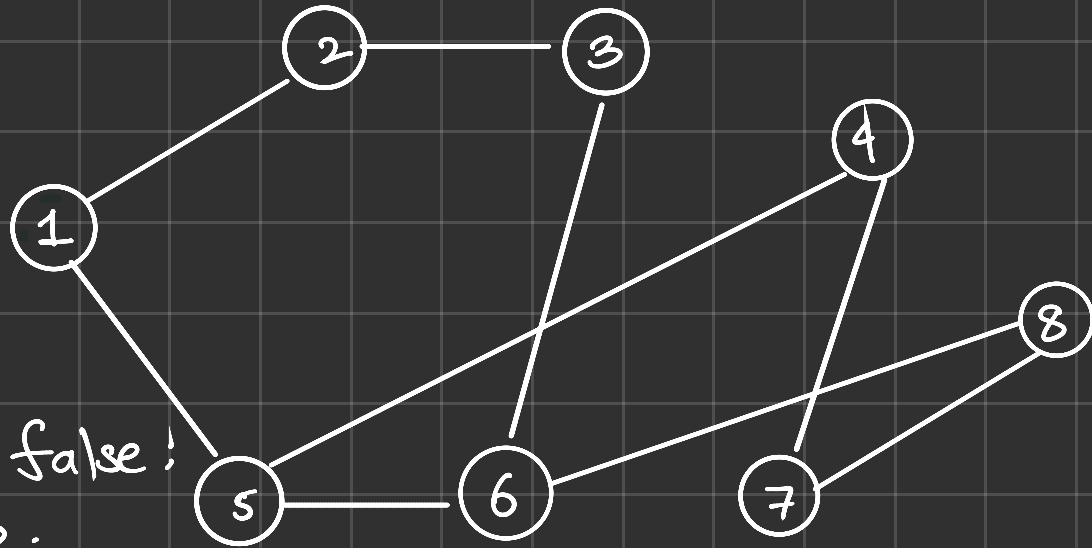
```

Implementing BFS

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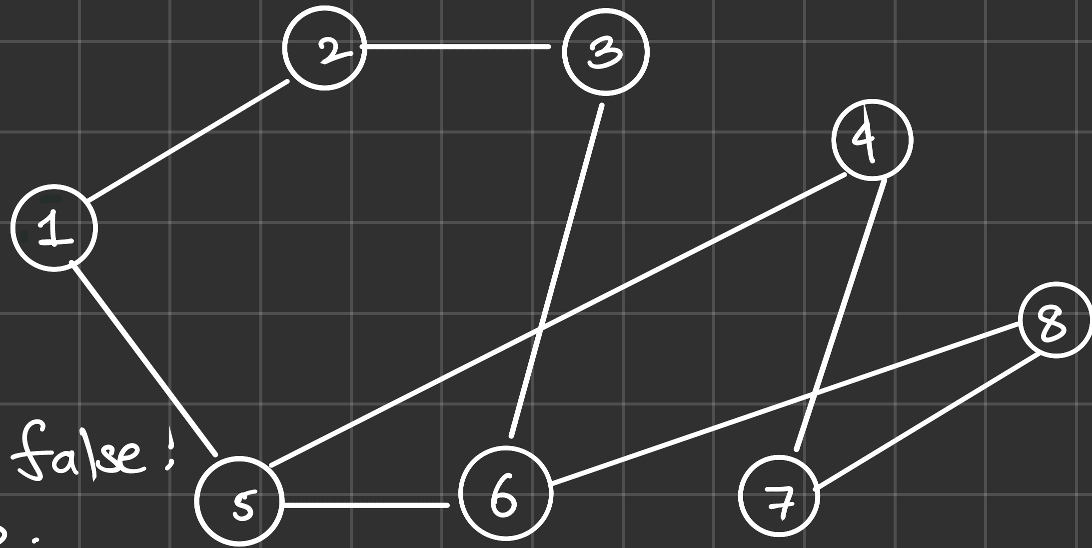
            }
        }
    }
}
```

1	2	3	4	5	6	7	8
f	f	f	f	f	f	f	f

discovered

Implementing BFS

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        }
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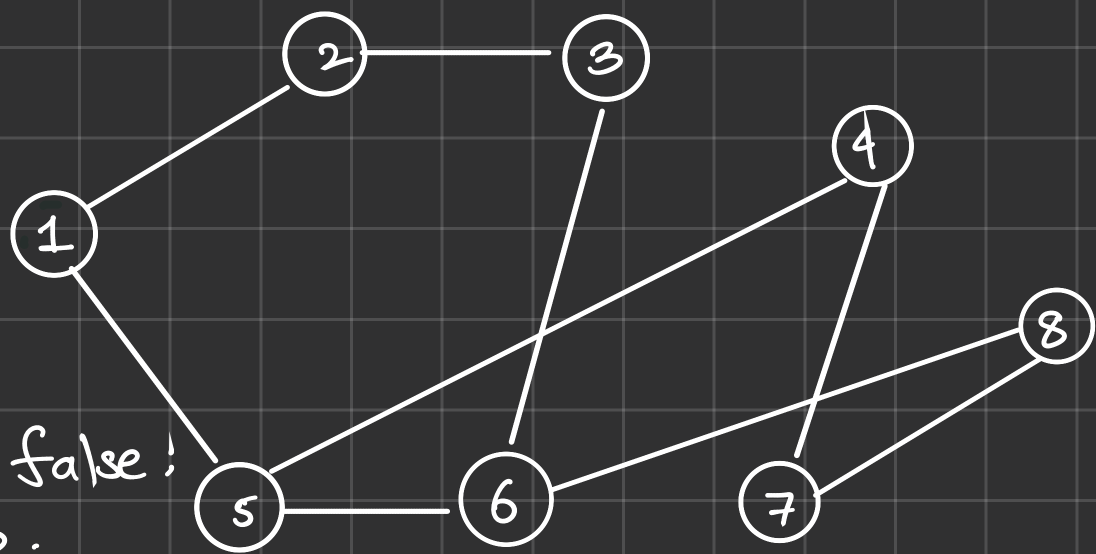
}
```

1	2	3	4	5	6	7	8
t	f	f	f	f	f	f	f

discovered

Implementing BFS

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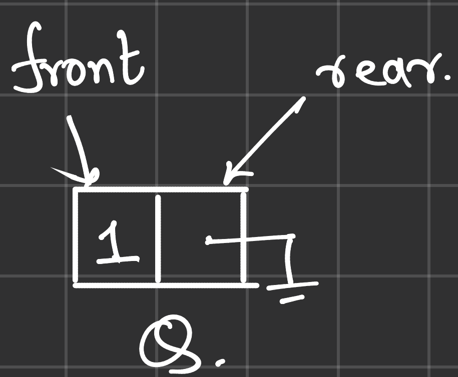
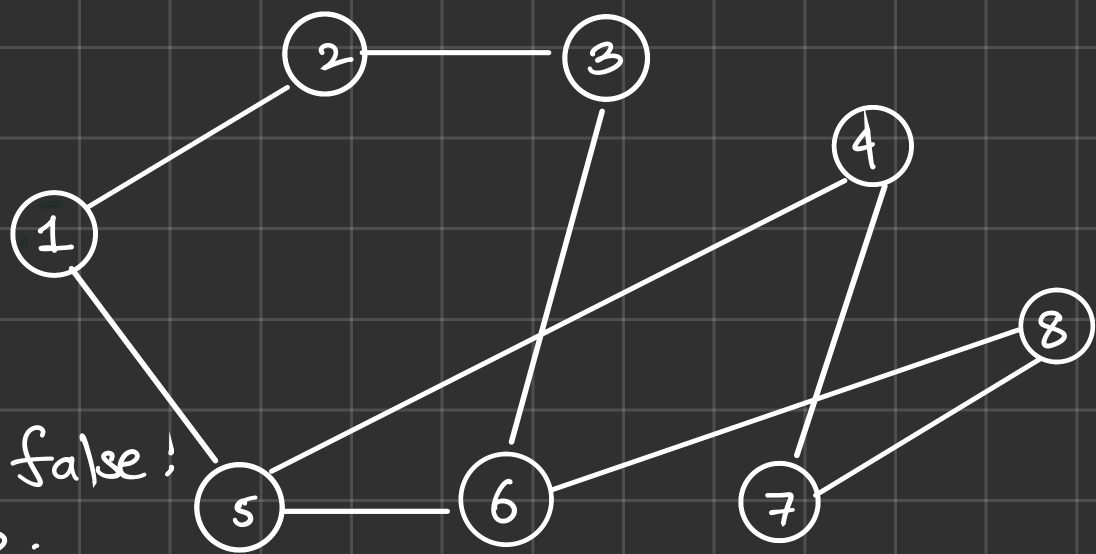
1	2	3	4	5	6	7	8
t	f	f	f	f	f	f	f

discovered



Implementing BFS

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```

1	2	3	4	5	6	7	8
t	f	f	f	f	f	f	f

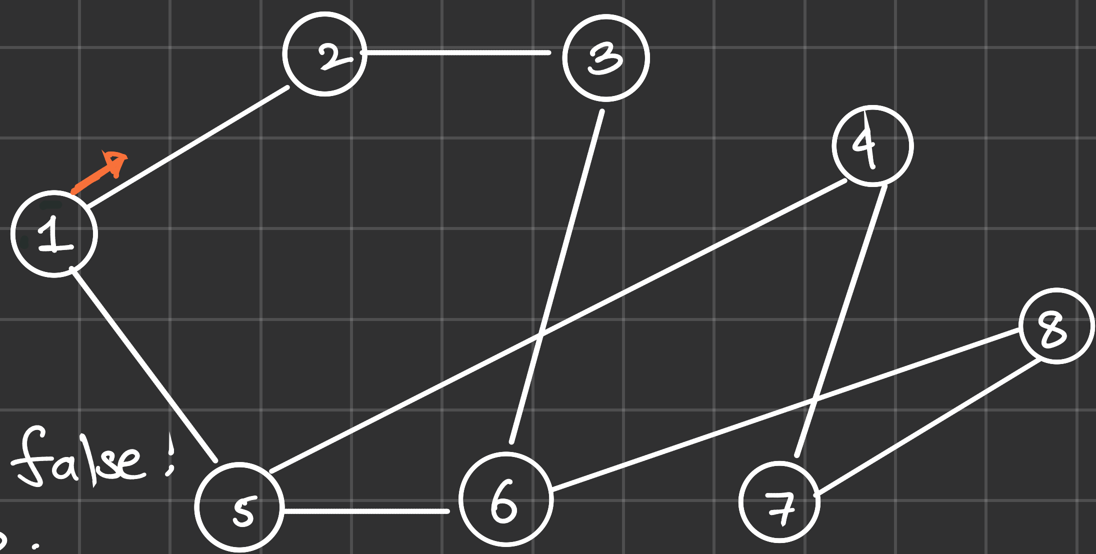
discovered

null

Q.

$v \leftarrow 1$

Implementing BFS

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    }
}
```

1	2	3	4	5	6	7	8
t	f	f	f	f	f	f	f

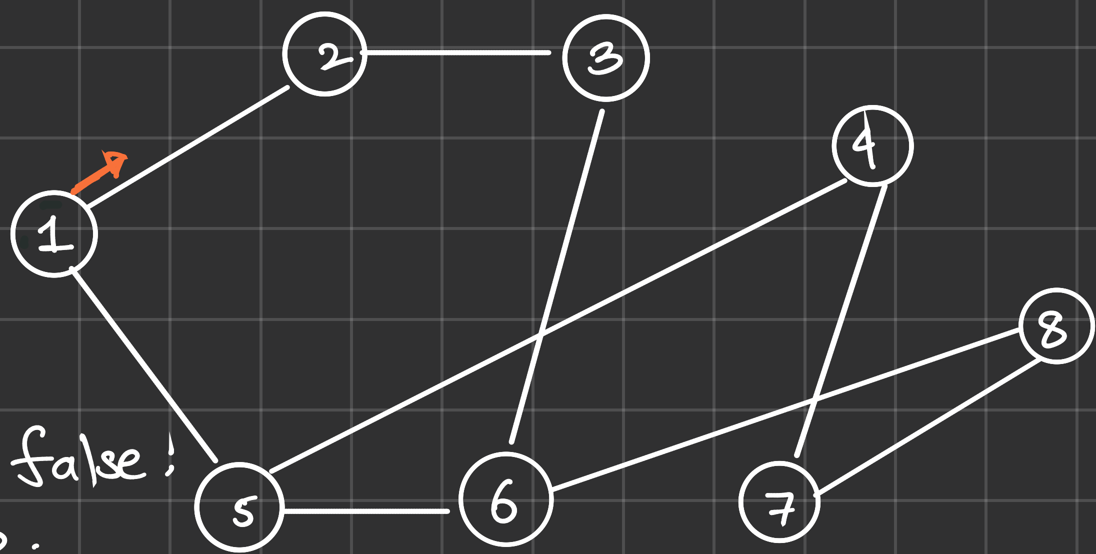
discovered

null.

Q.

$v \leftarrow 1$

Implementing BFS

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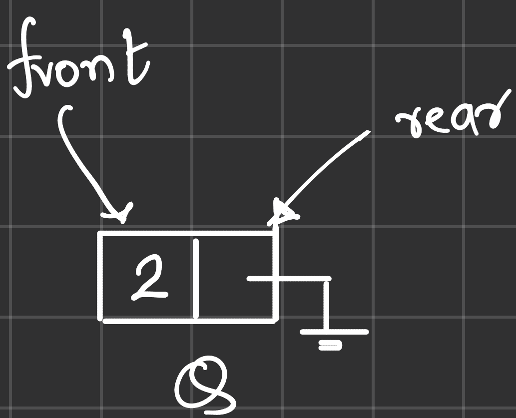
```

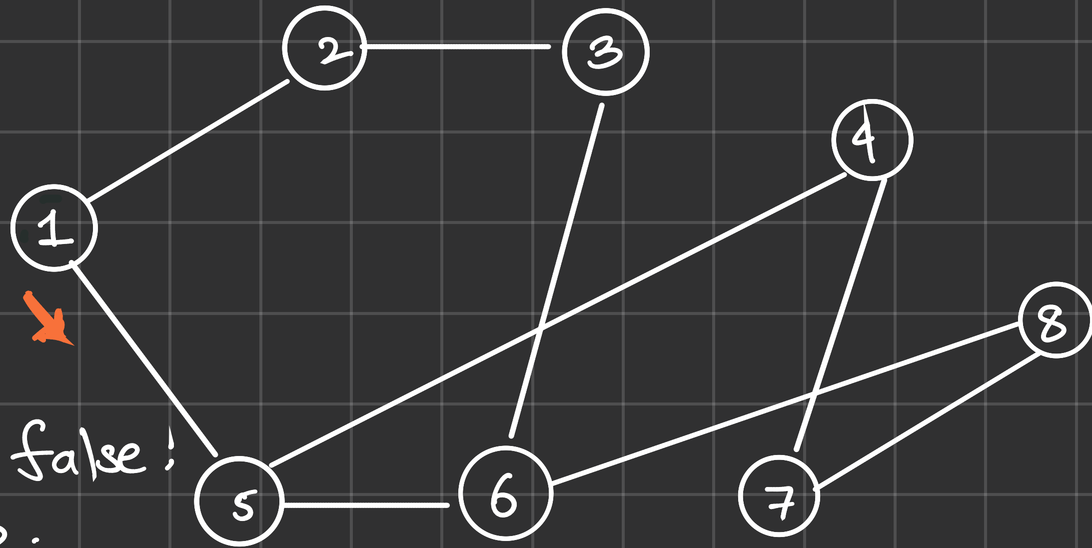
1	2	3	4	5	6	7	8
t	t	f	f	f	f	f	f

discovered



$v \leftarrow 1$

Implementing BFS

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        }
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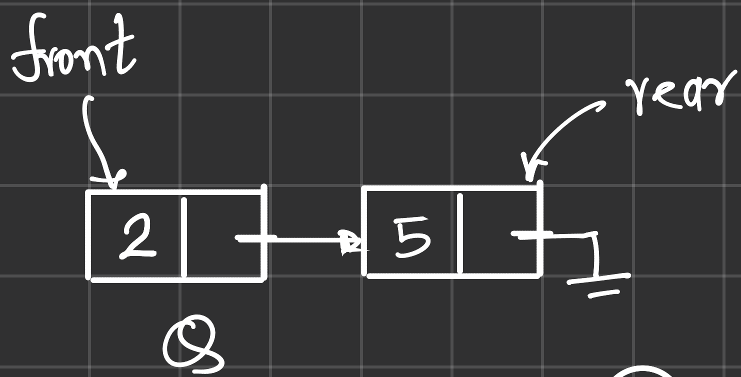
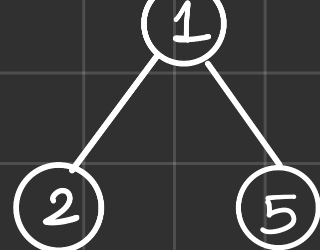
}
```

1	2	3	4	5	6	7	8
t	t	f	f	t	f	f	f

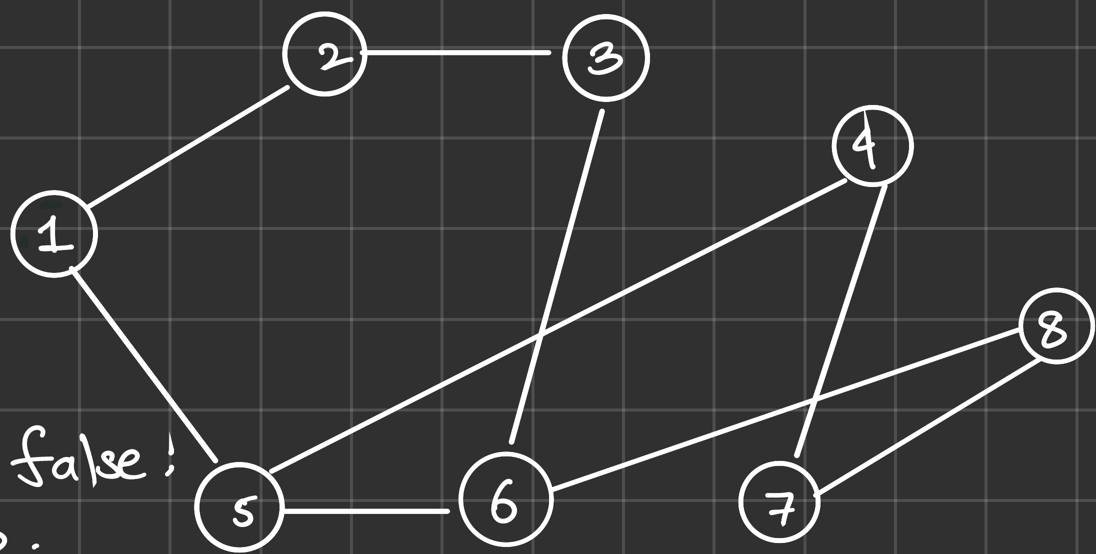
discovered



$v \leftarrow 1$



Implementing BFS



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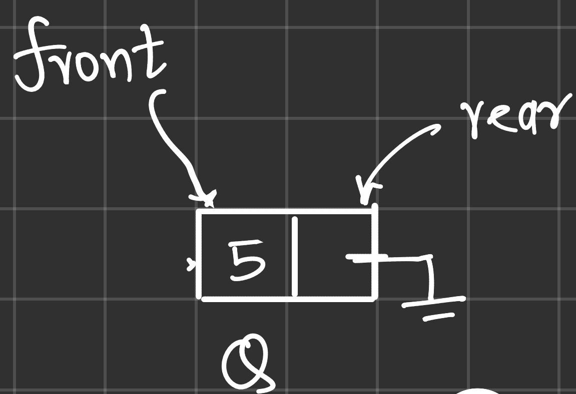
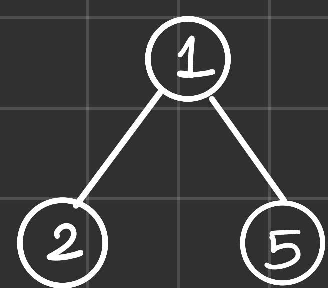
            }
        }
    }
}
```

1	2	3	4	5	6	7	8
t	t	f	f	t	f	f	f

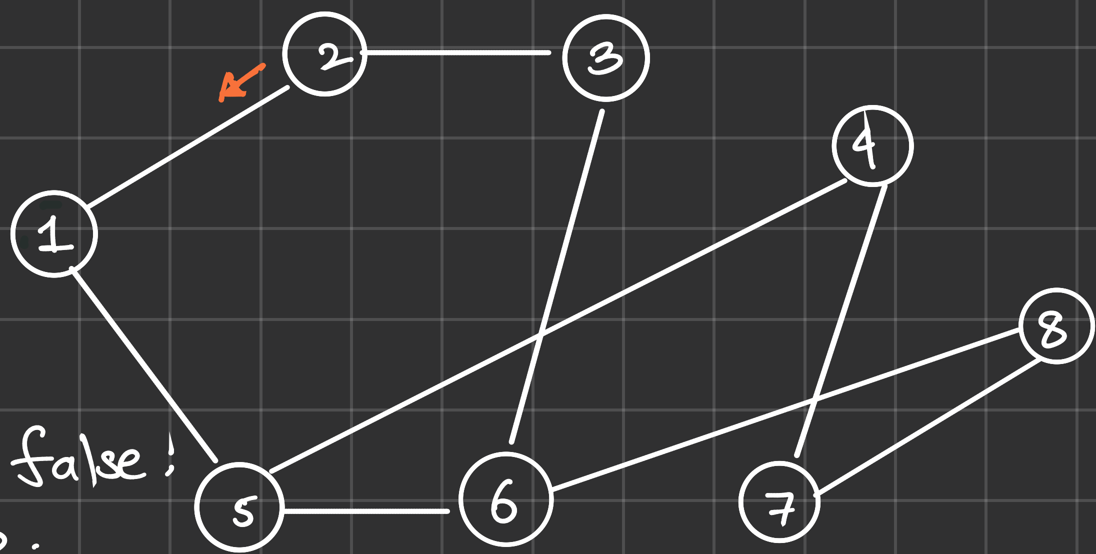
discovered





v ← 2

Implementing BFS

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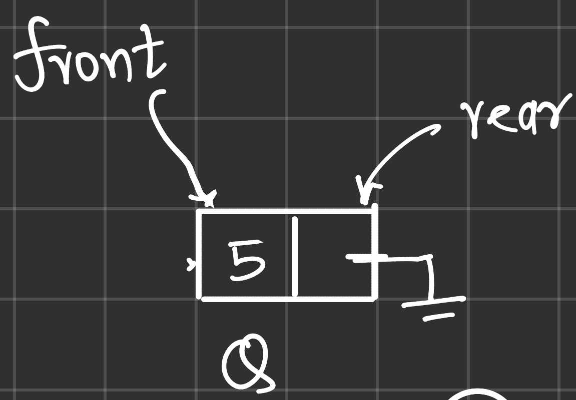
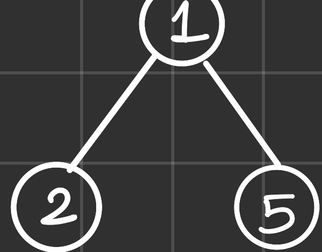
            }
        }
    }
}
```

1	2	3	4	5	6	7	8
t	t	f	f	t	f	f	f

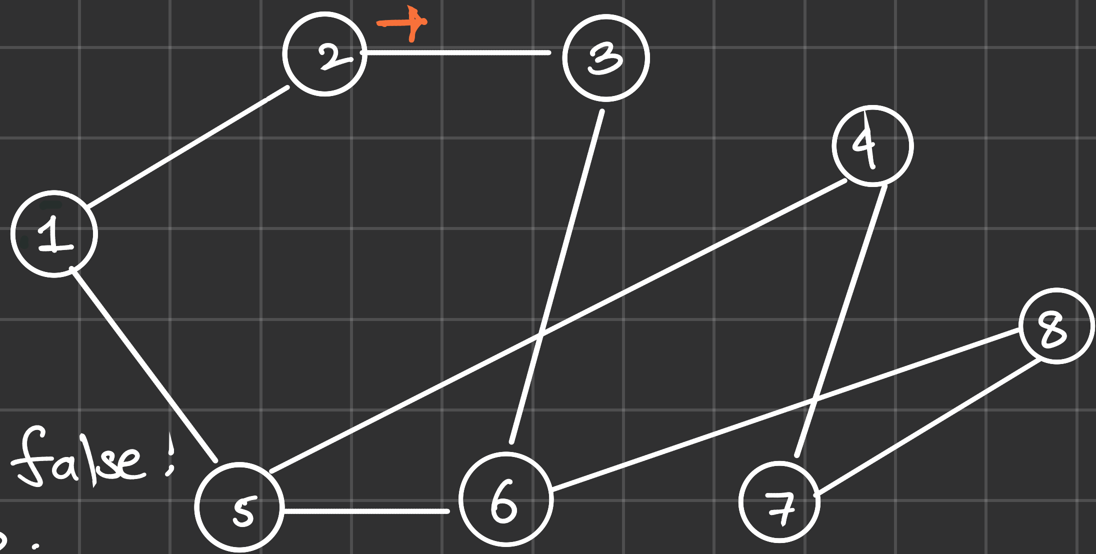
discovered





$v \leftarrow 2$

Implementing BFS

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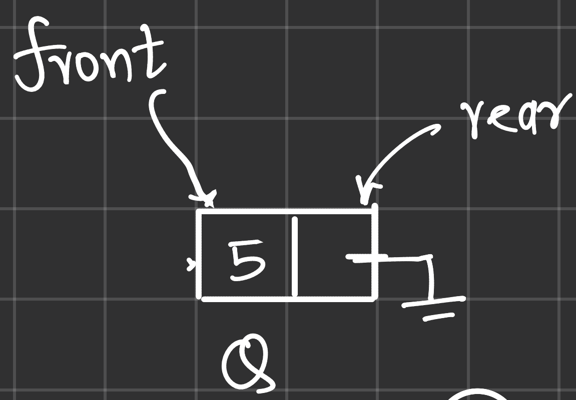
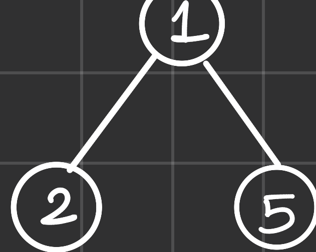
                add (v,w) to the BFS tree;
            }
        }
    }
}
```

1	2	3	4	5	6	7	8
t	t	f	f	t	f	f	f

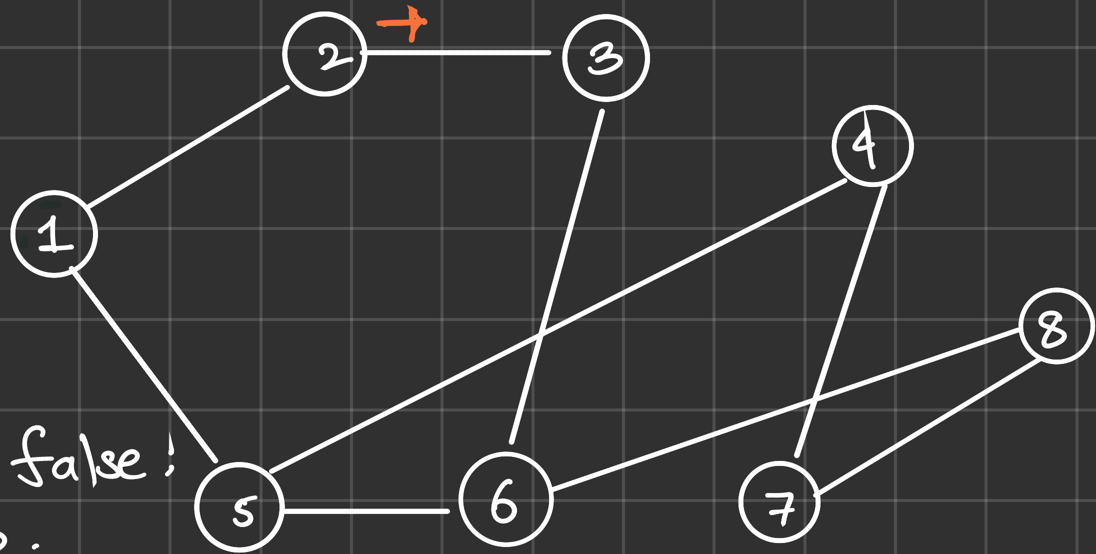
discovered





v ← 2

Implementing BFS

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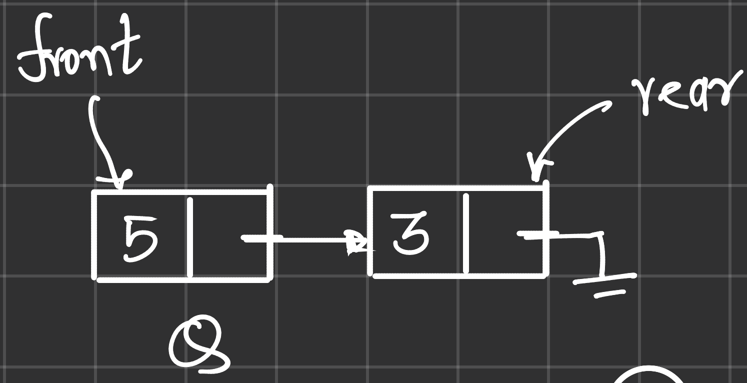
```

1	2	3	4	5	6	7	8
t	t	t	f	t	f	f	f

discovered



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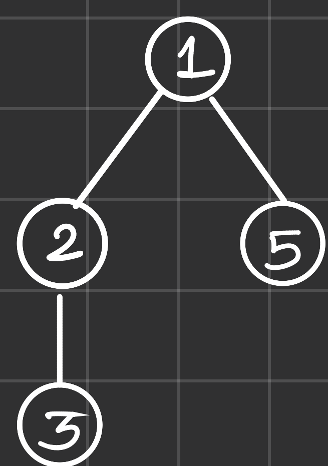

Implementing BFS

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                add (v,w) to the BFS tree;
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}
```

Running time :

Implementing BFS

```
BFS(s)
{   for each v ∈ V
    {   discovered[v] ← false;              } O(n)
    }
    discovered[s] ← true;                   } O(1)
    Q.enqueue(s)
    while (Q is not empty)
    {   v ← Q.deque();                      } O(1)
        for each neighbor w of v
        {   if (discovered[w] = false)
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Running time :

Implementing BFS

```
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    }
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    Q.enqueue(s)
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            {
                discovered[w] = true ; Q.enque(w);          O(d(v))
                add (v,w) to the BFS tree;
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    }
}
```

Running time :

Implementing BFS

```
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    }
    discovered[s] ← true;                      O(1)
    Q.enqueue(s)
    while (Q is not empty)
    {   v ← Q.deque();                         O(1)
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            {   discovered[w] = true ;  Q.enque(w);        O(d(v))
                add (v,w) to the BFS tree;
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```

Running time :

Main Observation : Each vertex is discovered only once
$\Rightarrow$ Each vertex enters the queue only once.

Implementing BFS

```
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{   for each v ∈ V
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```

Running time : $O(n) + O(1) + \sum_{v \in V} O(1 + d(v))$

Main Observation : Each vertex is discovered only once
$\Rightarrow$ Each vertex enters the queue only once.

Implementing BFS

```
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{   for each v ∈ V                                   } O(n)
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    }
    discovered[s] ← true;                            } O(1)
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    while (Q is not empty)
    {   v ← Q.deque();                               } O(1)
        for each neighbor w of v
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Running time : $O(n) + O(1) + \sum_{v \in V} O(1 + d(v))$

$= O(n) + O(1) + O(n+m)$

$= O(n+m)$

Implementing BFS

```
BFS(s)
{   for each v ∈ V
    {
        discovered[v] ← false;
    }                                          } O(n)
    discovered[s] ← true;
                                               } O(1)
    Q.enqueue(s)
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    {   v ← Q.deque();                         } O(1)
        for each neighbor w of v
        {   if (discovered[w] = false)
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                discovered[w] = true ;  Q.enque(w);
                add (v,w) to the BFS tree;     } O(d(v))
            }
        }
    }
}
```

Running time : $c_1 n + c_2 + \sum_{v \in V} c_3 + c_4 d(v)$

$= c_1 n + c_2 + c_3 \cdot n + c_4 \cdot 2m$

$= O(n+m)$

Our problem: Find if t is reachable from s.

```
Reachable(s,t)
{
```

Our problem : Find if t is reachable from s.

```
Reachable(s,t)
{
    BFS (s) ;
    if ( discovered[t] = true)
        t is reachable from s
    else
        t is not reachable from s
}
```

Problem: Given a connected graph G, find if it is a bipartite graph?

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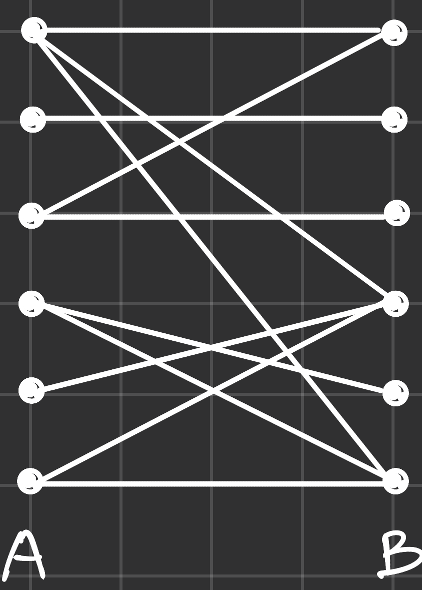
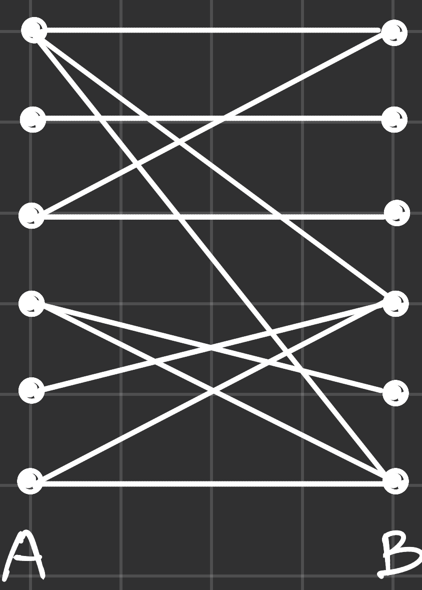


For each edge (x,y) in a bipartite graph,
$(x \in A$ and $y \in B)$ or $(x \in B$ and $y \in A)$

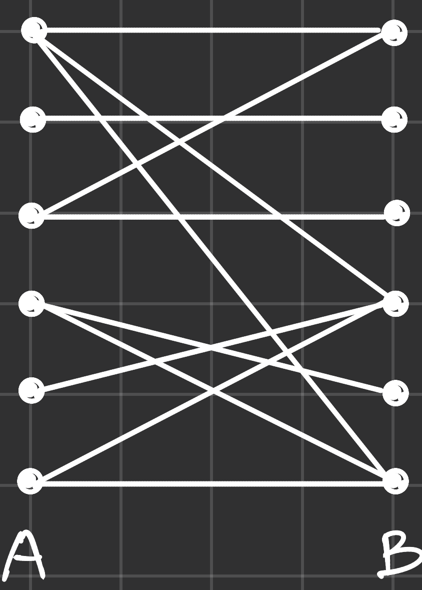
Problem : Given a connected graph G, find if it is a bipartite graph?



For each edge (x,y) in a bipartite graph, $(x \in A$ and $y \in B)$ or $(x \in B$ and $y \in A)$

Property : If graph G is bipartite, it cannot contain odd cycle

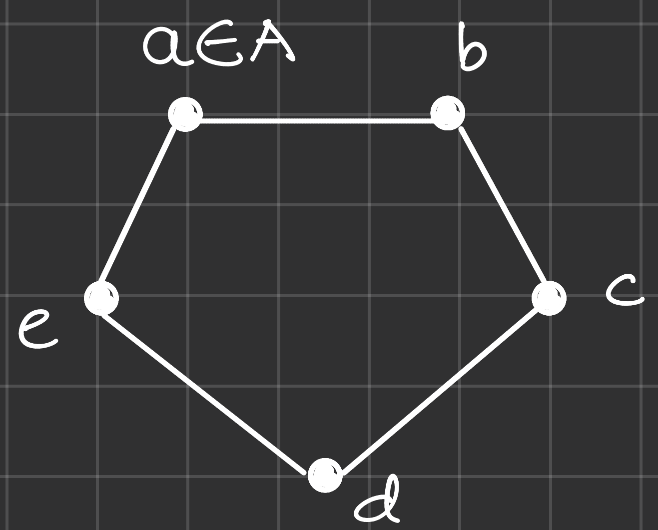
Problem : Given a connected graph G, find if it is a bipartite graph ?



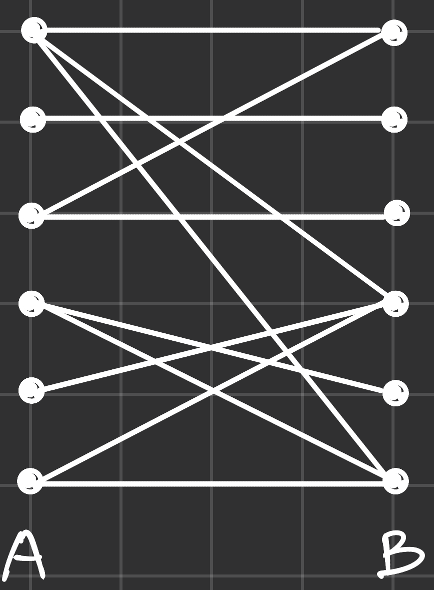
For each edge (x,y) in a bipartite graph, $(x \in A$ and $y \in B)$ or $(x \in B$ and $y \in A)$

Property : If graph G is bipartite, it cannot contain odd cycle

Proof : By contradiction



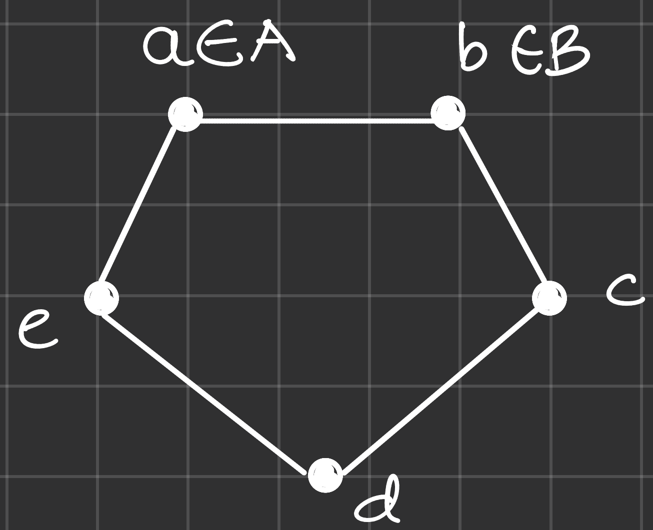
Problem: Given a connected graph G, find if it is a bipartite graph?



For each edge (x, y) in a bipartite graph, $(x \in A$ and $y \in B)$ or $(x \in B$ and $y \in A)$

Property: If graph G is bipartite, it cannot contain odd cycle

Proof: By contradiction



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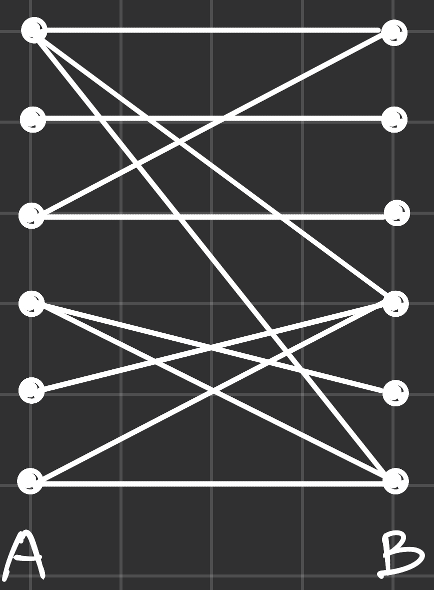

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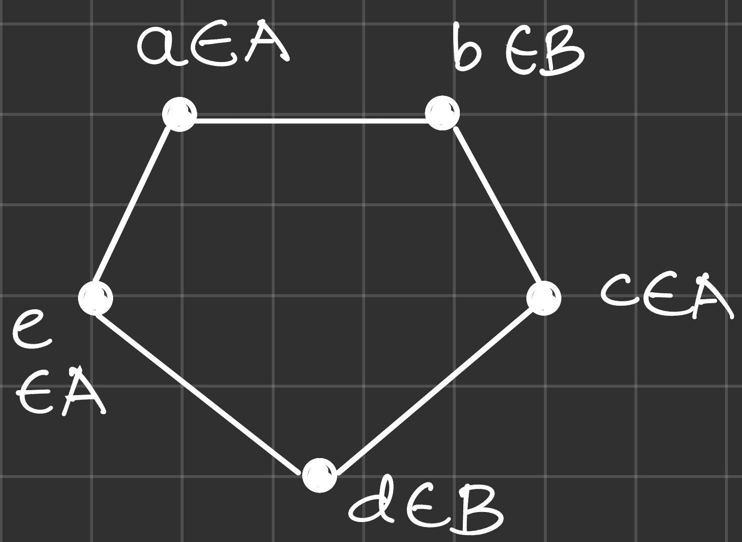

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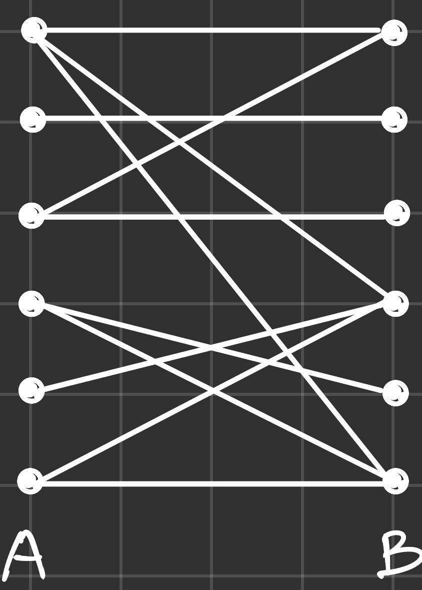

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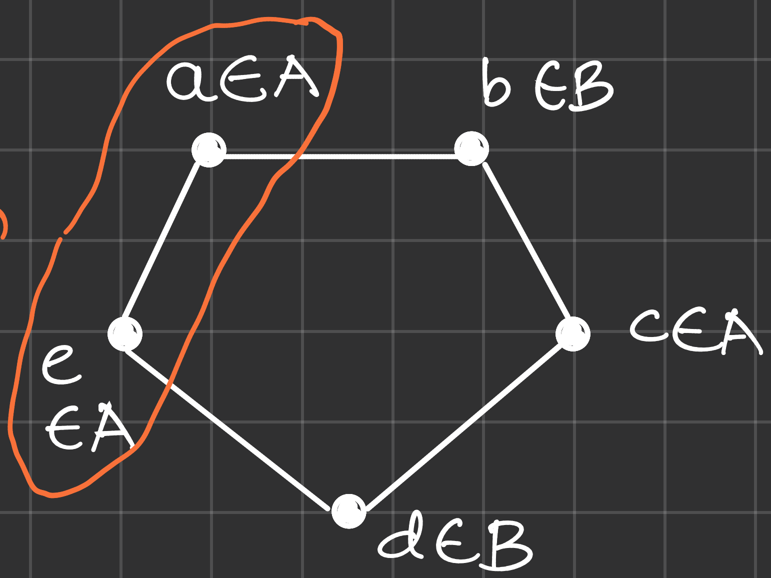

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Is this property is true?

If a graph has no odd cycle, then it is bipartite.

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An algorithm is used as the proof.

BFS algorithm

Algo

Pick an arbitrary vertex v;

BFS(v);

If there exists an edge xy s.t x & y are at same level in the BFS tree

{

We have found an odd cycle

$\Rightarrow$ G is not bipartite

}

else

{

We have found a bipartite graph

$A = \{L_0, L_2, L_4, \ldots\}$ even layer vertices

$B = \{L_1, L_3, L_5, \ldots\}$ odd layer vertices

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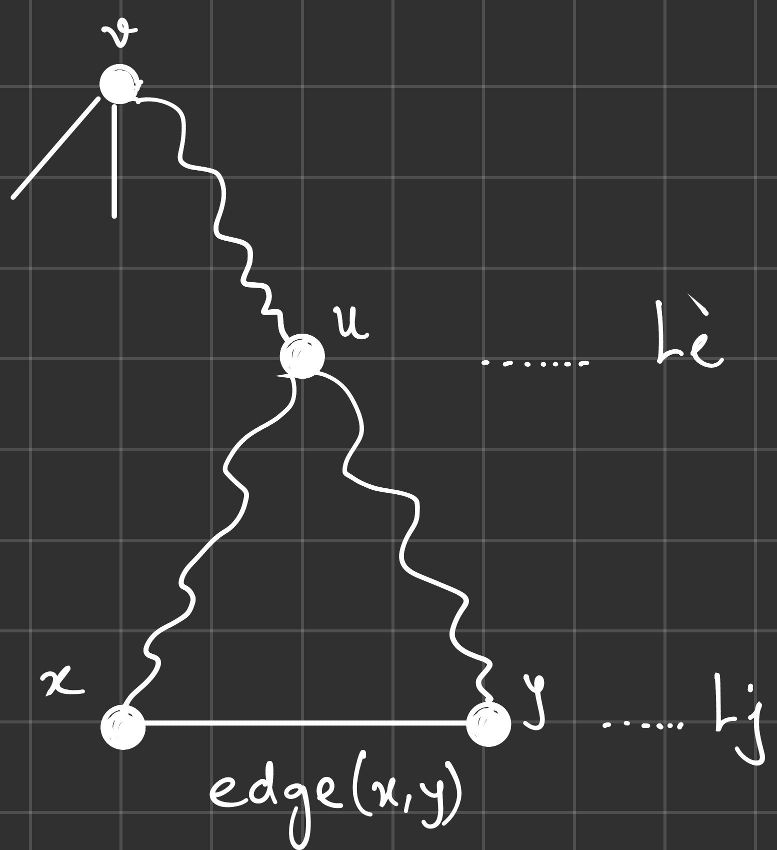

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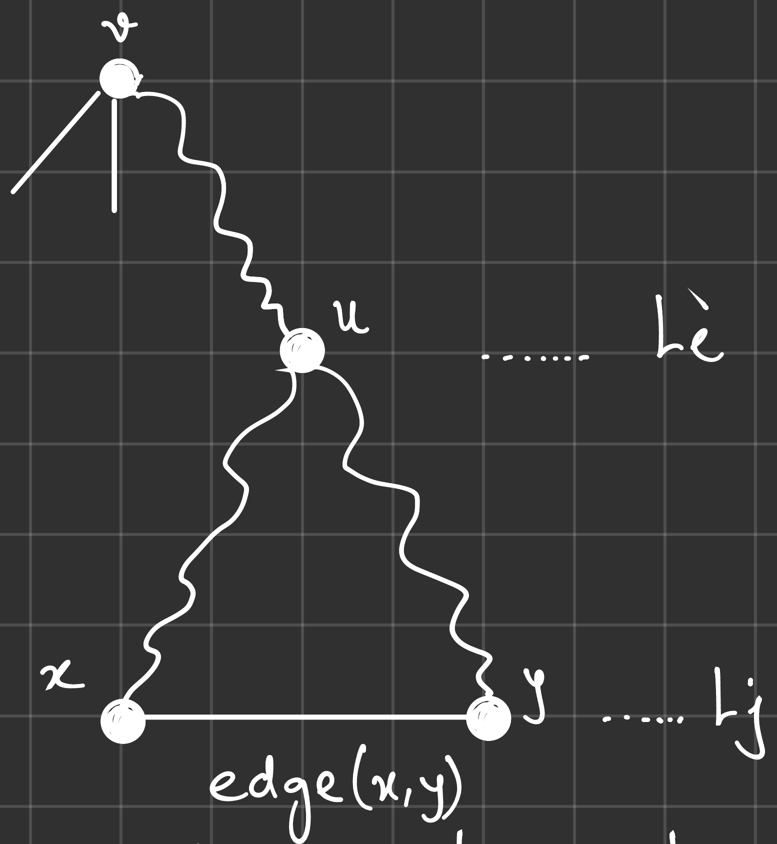

Length of the cycle $= |u \rightsquigarrow y| + 1 + |x \rightsquigarrow u|$

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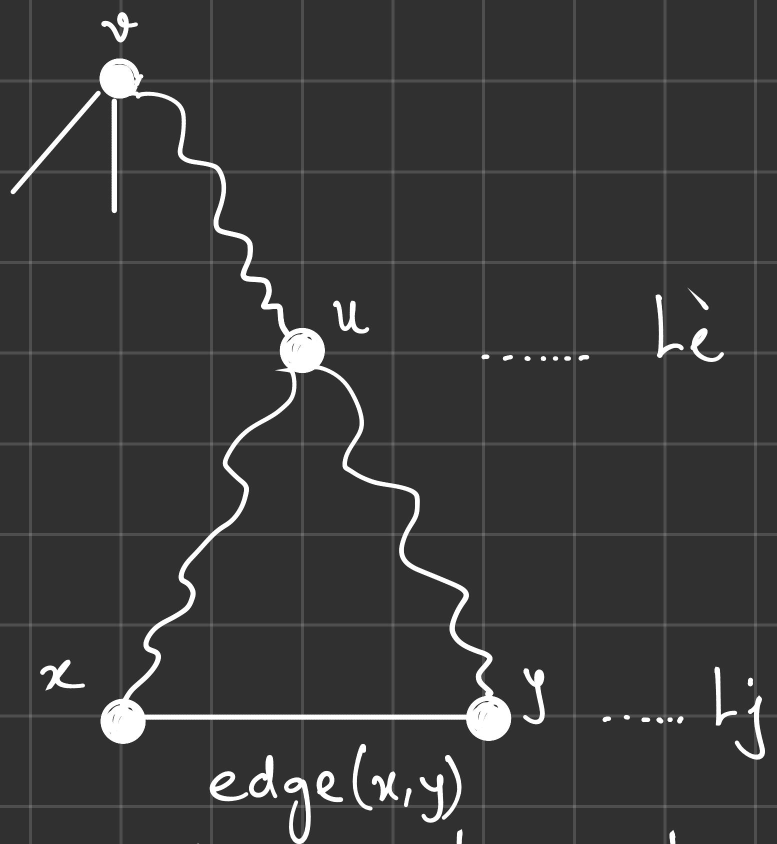

$$
\begin{aligned}
\text{Length of the cycle} &= |u \rightsquigarrow y| + 1 + |x \rightsquigarrow u| \\
&= (j-i) + 1 + (j-i) \\
&= 2(j-i) + 1
\end{aligned}
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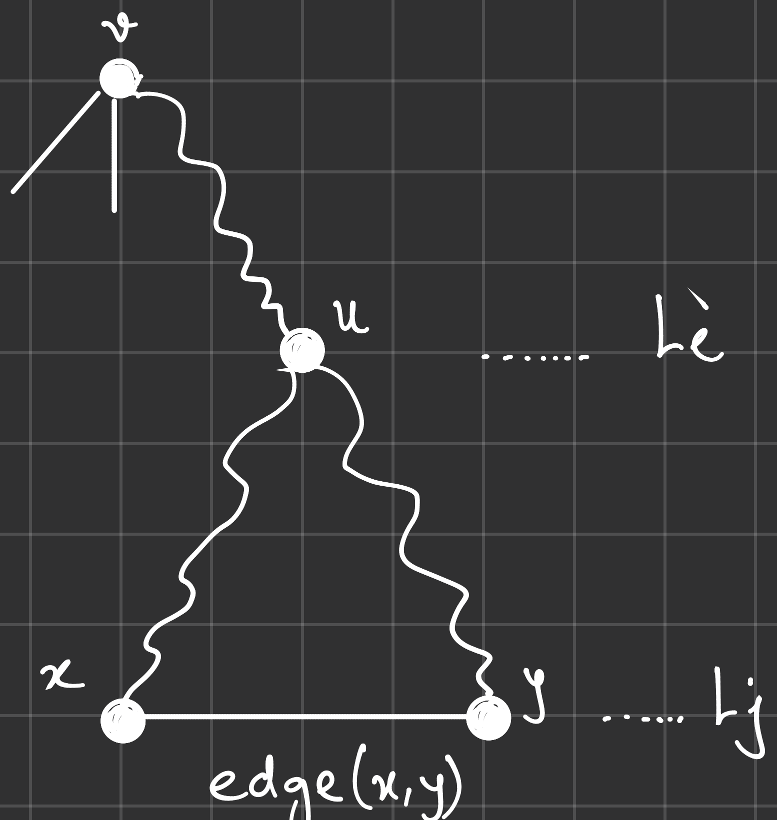

Length of the cycle $= |u \rightsquigarrow y| + 1 + |x \rightsquigarrow u|$

$= (j-i) + 1 + (j-i)$

$= 2(j-i) + 1$

odd

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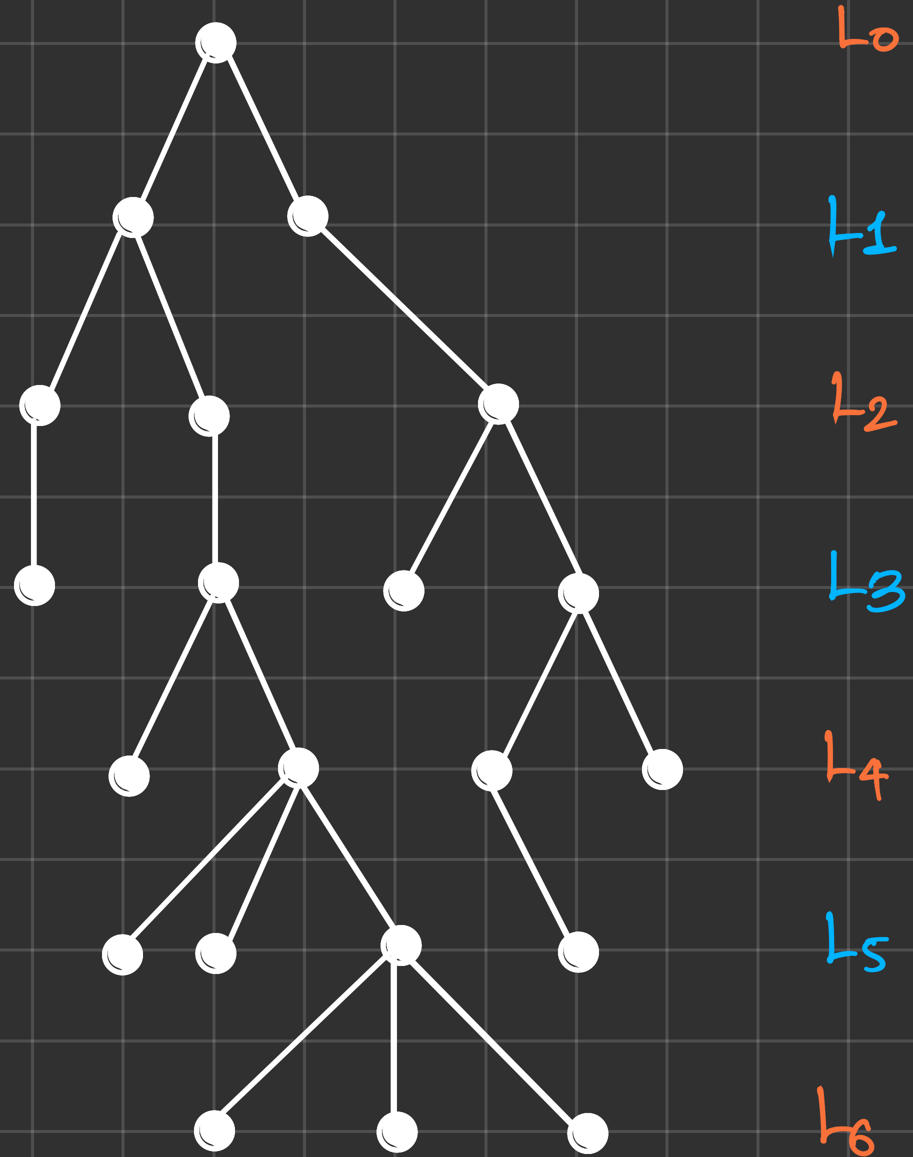

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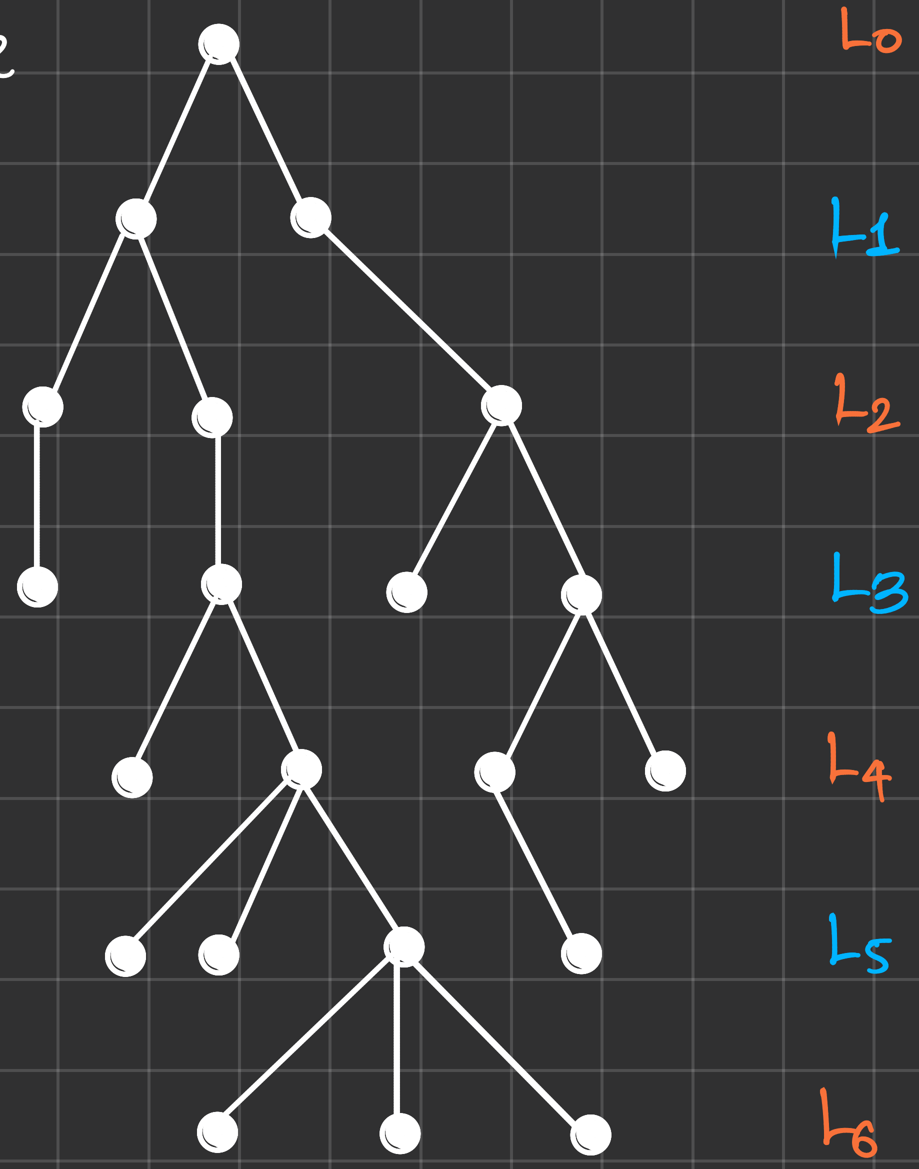

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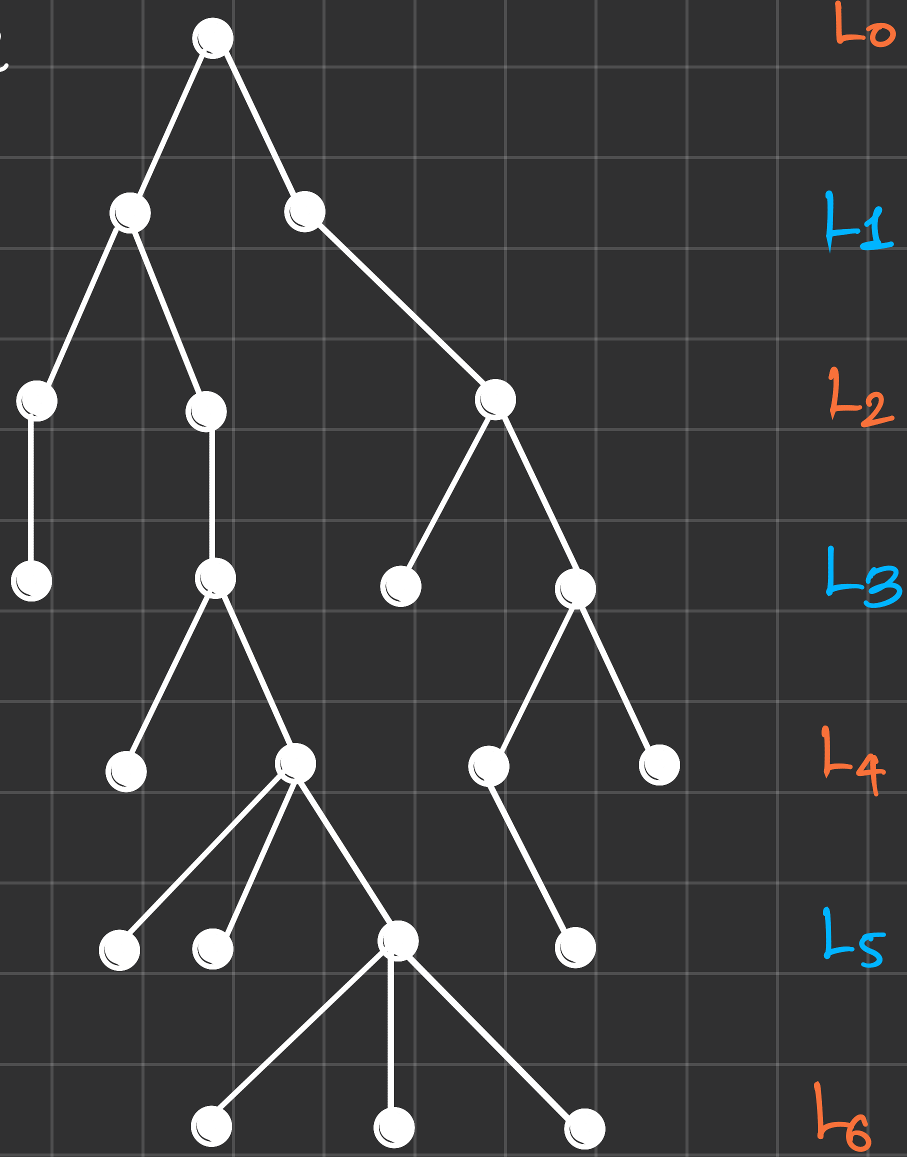
(1) If (x,y) is a tree edge



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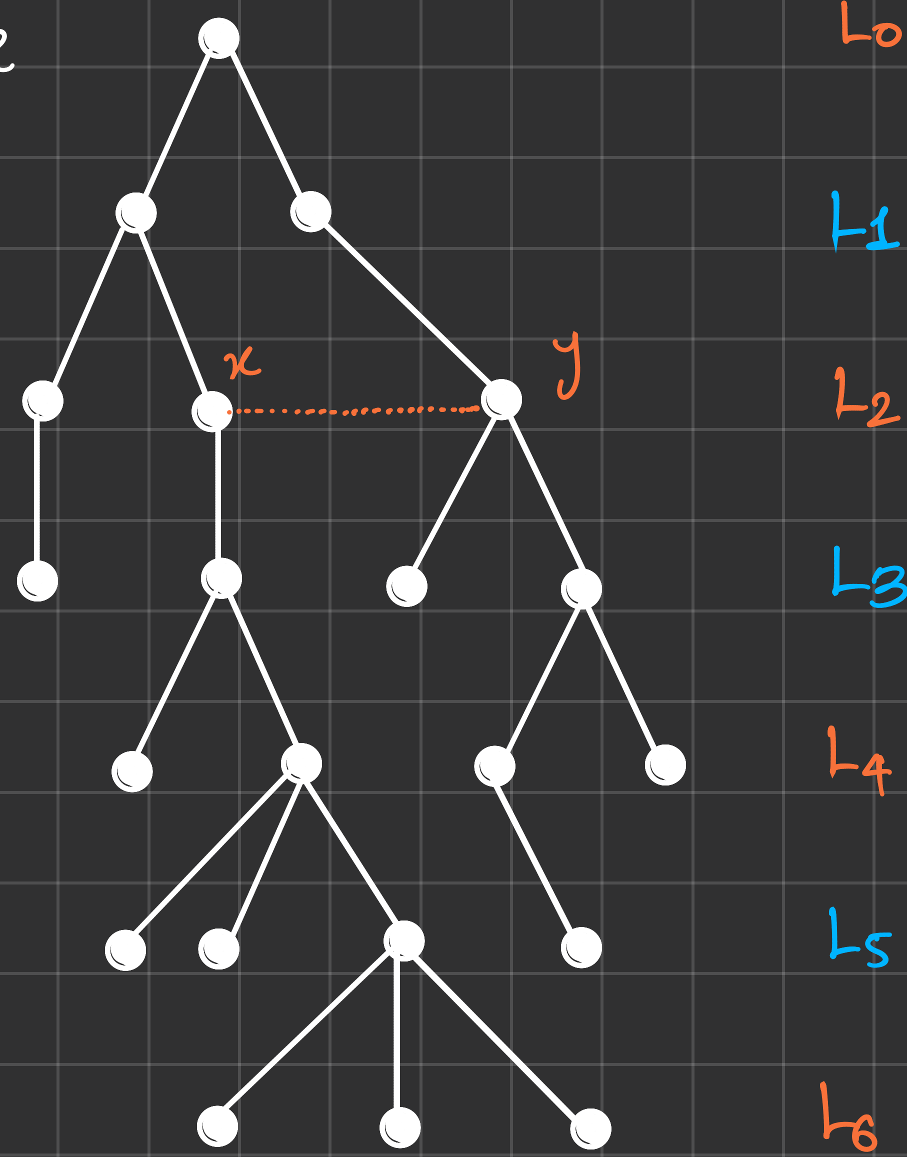

(1) If (x,y) is a tree edge
trivially true

(2) If (x,y) is a non-tree edge

(a) edge at same level

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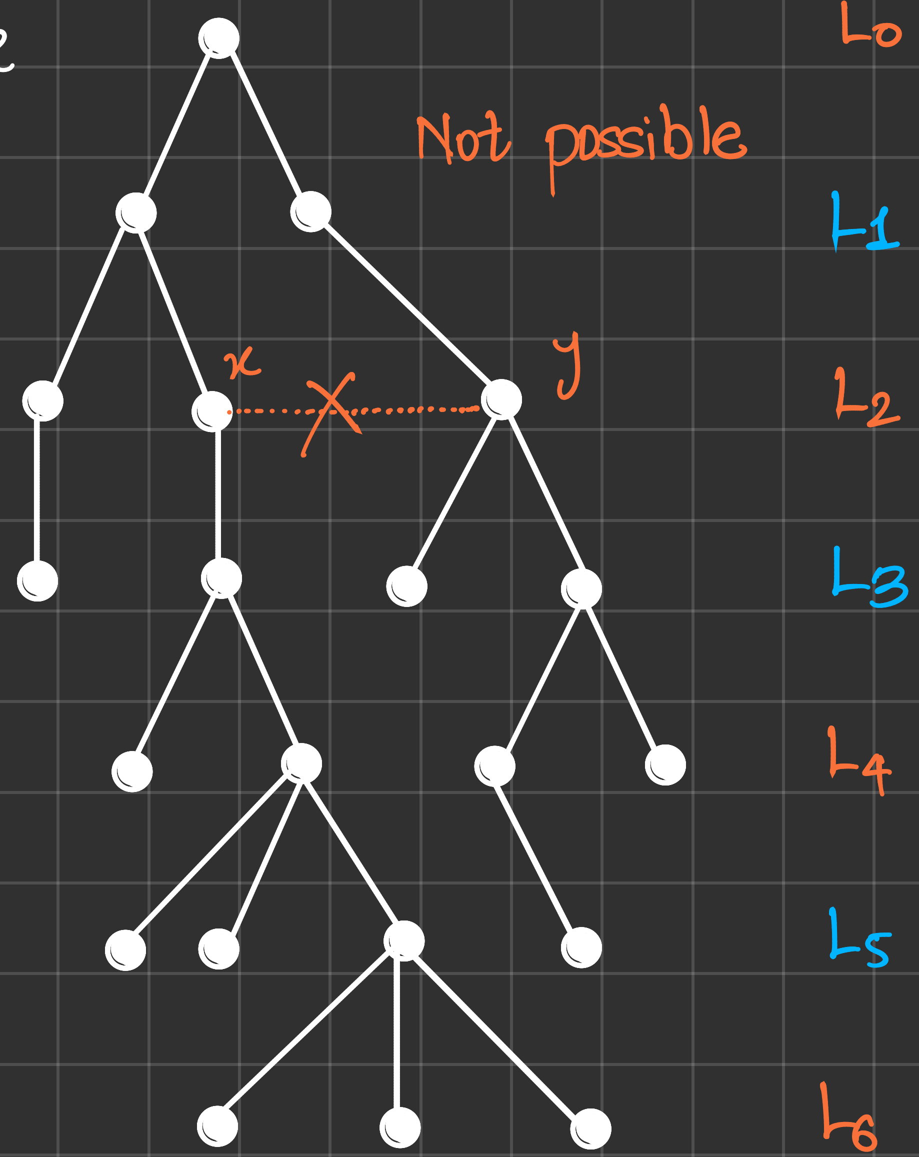

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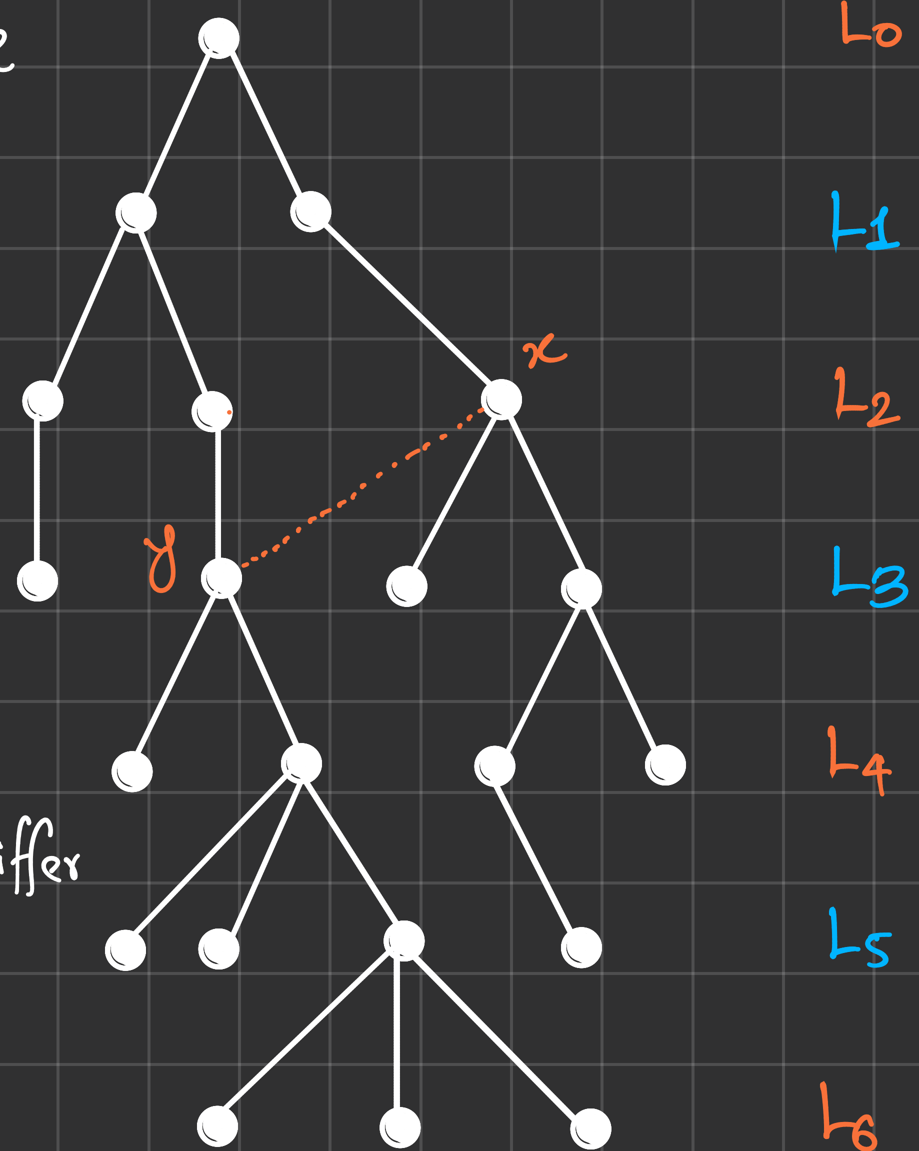

(1) If (x,y) is a tree edge trivially true

(2) If (x,y) is a non-tree edge

~~(a) edge at same level~~

(b) edge whose endpoint differ by 1 level

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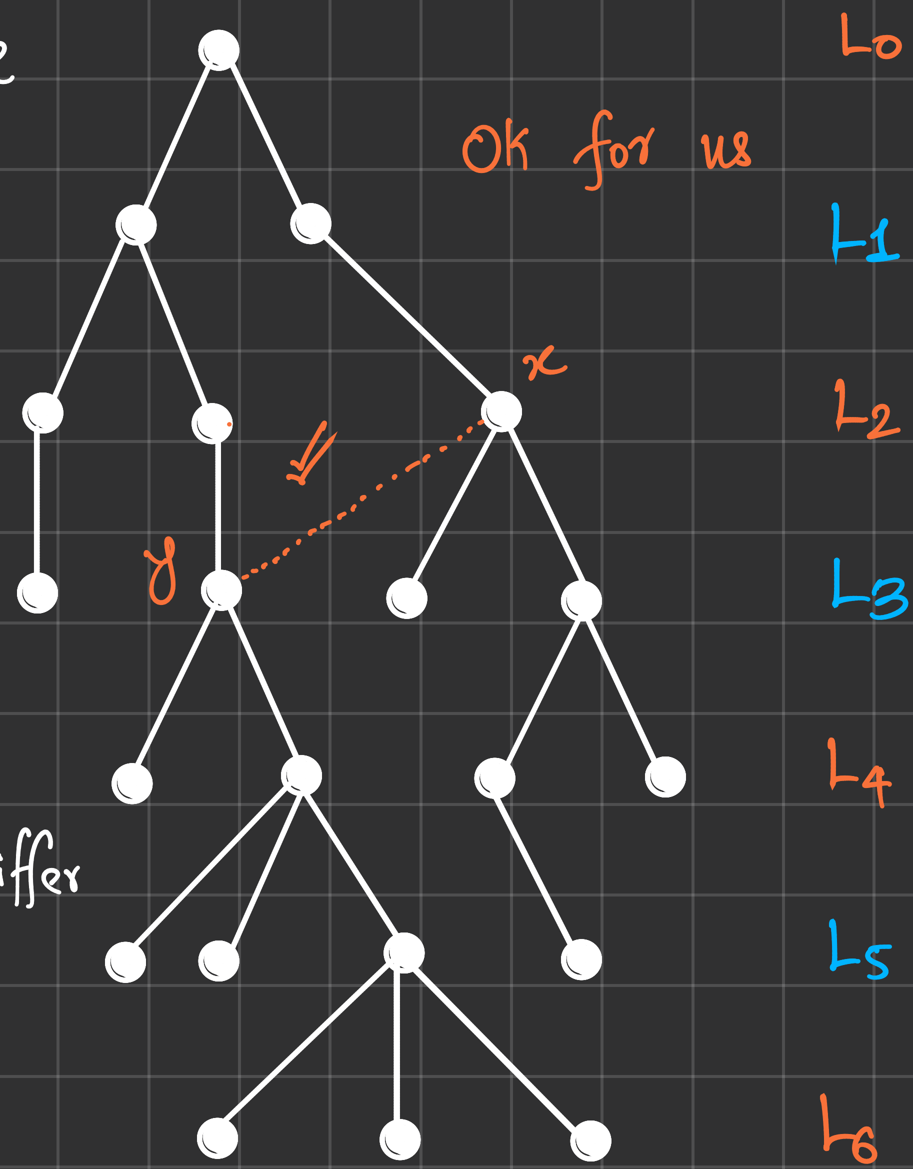

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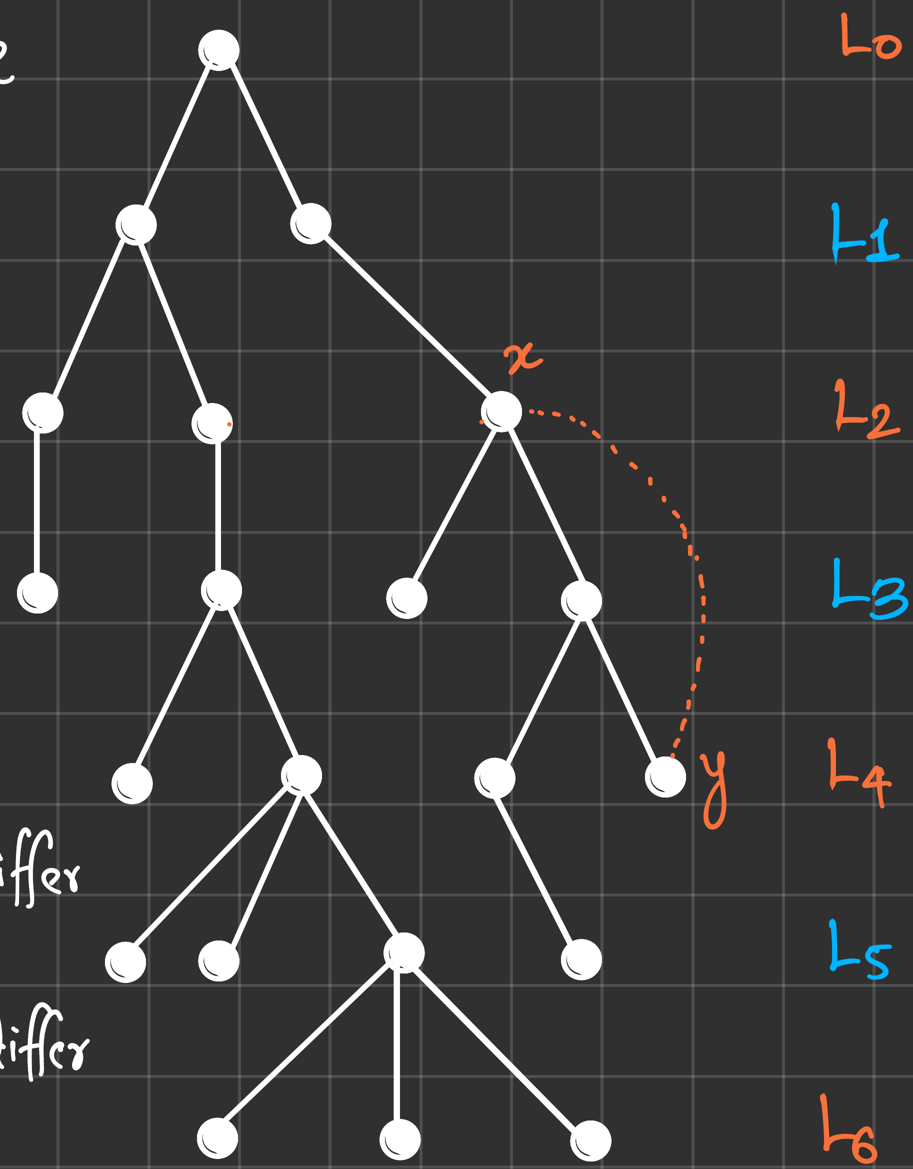

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(c) edge whose endpoint differ by >1 level

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