

Lab 3

August 26, 2021

For each question in this lab, please use the concept of heap that we have done in the class.

1. HeapSort

You are given a sequence of integers that implicitly represent a min-heap. Sort them using the heapsort algorithm.

Input: The first input line contains integer n ($1 \leq n \leq 1000$). In the second line, n numbers are separated by a space (all numbers are ≤ 1000). The sequence of numbers represents a min-heap.

Output: n numbers written in increasing order separated by a space.

```
Input   : 8
         1 3 5 4 10 6 7 11
Output  : 1 3 4 5 6 7 10 11
```

2. Is this a heap?

Given a sequence of integers implicitly representing a fully binary tree, check if it is a min-heap.

Input: The first input line contains integer n ($1 \leq n \leq 1000$). The second line n numbers separated by a space (all numbers are ≤ 1000).

Output: Yes, if the sequence implicitly represents a min-heap. No, otherwise.

```
Input   : 5
         1 3 5 4 6
Output  : Yes

Input   : 5
         1 7 5 8 6
Output  : No
```

3. Two lowest numbers

You are given a sorted array of numbers. In this array, we perform the following operation. Take the two lowest numbers, say x is the minimum, and y is the second minimum. Remove x and y and replace them by $x + 2y$ in the array. Keep repeating this operation until all the array numbers are $\geq k$ (where k is a parameter). You have to find the number of operations in which you can achieve this goal.

Input: The first input line contains two integers n and k ($1 \leq n, k \leq 1000$). The second line contains n numbers separated by a space. The numbers are sorted, and each number is ≤ 1000 .

Output: The total number of operations to be done such that all the numbers in the array are $\geq k$. If the goal cannot be achieved, then print -1.

```
Input   : 5 3
         1 1 1 1 1
Output  : 3
```

Explanation:

Iteration 1: 1 1 1 3

Iteration 2: 1 3 3

Iteration 3: 3 7

Input : 3 10
1 1 1

Output : -1

Explanation:

Iteration 1: 1 3

Iteration 2: 7

We cannot now perform any more operations.

4. Query Heap

In this question, you have to perform the following operations on a min-heap \mathcal{H} .

1. a x : add the number x to the heap \mathcal{H} .
2. d : Delete the minimum element from the heap.
3. q : print the minimum element of the heap.

Input: The first line of the input will be n ($n \leq 1000$) – the number operations to follow each in a separate line. The operation will be of the type given in the above enumeration.

Output: For each operation of type 'q', print the current minimum element of the min-heap. Each print should be on a new line.

Input : 5
a 1
q
a 2
d
q
Output : 1
2

5. k -th smallest element

Assume that you are given a sequence of n integers. For any prefix of this sequence of size $\geq k$, where k is a parameter, we need to find the k -th smallest element in the prefix. For example, if the sequence is 3 4 1 5 and $k = 2$, then

1. For the first two numbers, 3 4, the k -th largest number is 4.
2. For the first three numbers, 3 4 1, the k -th largest number is 3.
3. For the first four numbers, 3 4 1 5, the k -th largest number is 3.

Your job is to print all the k -th smallest numbers for all possible sequence prefixes – in order. You should use a heap of size k to solve this problem.

Input: The first input line contains two integer n and k ($1 \leq n, k \leq 1000$). The second line contains n numbers separated by a space. Each number is ≤ 1000 .

Output: $n - k + 1$ numbers separated by a space. The first number represents the k -th smallest number of the first k numbers in the sequence. The last number represents the k -th smallest element of the entire sequence.

Input : 5 3
1 9 5 4 6
Output : 9 5 5