

Please Note

1. Please prove the running time and the correctness of the algorithm.
2. You can state and use any algorithm done in the class.
3. The exam is open notes.
4. Do not use the internet.
5. The total time for this exam is 1 hour 30 minutes.

ES242 – End Sem

November 23, 2021

- (3 points) You are given a linked list on n numbers. The numbers in the linked list are sorted. You need to write a function $\text{PUT}(\text{start}, x)$ that takes the starting node of the linked list and adds a node with value x in the linked list such that after the addition the list is still sorted. Write a correct implementation of this function (you may write pseudo code or C code). You can also assume that all the number are distinct.
- (5 points) You are given an *sorted* array A on distinct numbers that can contain negative numbers. An index i is called an *identical index* if $A[i] = i$. Design an algorithm that can find if A contains an identical index in $O(\log n)$ time.
- You are given a directed acyclic graph (DAG) G . A vertex v in the DAG is called an *important* vertex if there is a directed path from v to every other vertex in the graph. Your job is to find if G contains an *important* vertex. Design an algorithm that can find this in
 - (1 point) $O(mn)$ time.
 - (5 point) $O(m + n)$ time.
- (10 points) A binary tree is complete if its every internal node has two children, and every leaf has exactly the same depth. Design and analyze an algorithm to compute the depth of *largest complete subtree* of a given binary tree in $O(n)$ time.

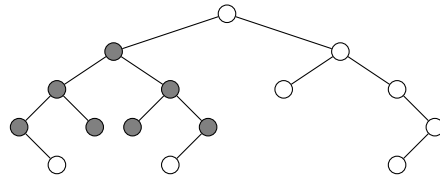


Figure 1: The depth of the largest complete subtree in the above binary tree is 2