Please Note

1. Please prove the running time and the correctness of the algorithm.
2. You can state and use any algorithm done in the class.
3. The exam is open notes.
4. Do not use the internet.

ES242 – Mid Sem

September 22, 2021

1. (5 points) In the class, we saw that in a Heap, we can add an element in $O(\log n)$ time and can report the minimum in $O(1)$ time (as it is at the top of the heap). Design a data-structure that can perform the following:

   ADD(x): Add an integer x into the data-strucuture.

   REPORT-MIN(): Return the minimum of all the elements added to the data-structure.

   Both the operations need to be done in $O(1)$ time. Assume that you know that the maximum number of integers that can be added to the data-structure is $\leq n$ where n is a parameter.

2. (5 points) Solve the following recurrence:

$$T(n) = T(n/2) + T(n/4) + n, \text{ for } n > 5$$
$$T(n) = 1, \text{ for } n \leq 5$$

3. (5 points) You are given an array A of integers. For each suffix $A[1 \dots i]$ of the array, you have to output the k-th smallest (here k is a parameter) integer in this subarray where $k \leq i \leq n$. Design an algorithm that performs this task in $O(n \log k)$ time.

4. (10 points) You are given access to an in-place sorting algorithm SQSORT(k) which sorts all elements in an array A from index $k+1$ to $k+\sqrt{n}$, that is it sorts the array $A[k+1 \dots k+\sqrt{n}]$ (we can assume that $\sqrt{n}$ is an integer). We want to use just this function SQSORT() and sort the array, that is, we are not allowed any other comparison, swap, move, copy inside the array. How many times will you call the function SQSORT(k) to sort the array $A[1 \dots n]$?